Global Author and Four Co-Authors

Educational & Didactic Communication 2014

Vol.1 Application of Quantitative Research Degrees

Methodology of Scientific Research

- Degrees of Quantitative Research - Part A
- Algorithm of Quantitative Research - Part A

Illustration of Qualitative Research Degrees - Part A

Illustration of Quantitative Research Degrees - Part A

Quantitative Research within Financial Markets - Part B, Part E

Quantitative Research within Magnetic Resonance - Part C

Quantitative Research within Aviation Safeguards - Part D

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The publisher: Publishing House DIDAKTIS
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Global Author of Monograph and Its Conception:
Assoc. Prof. RNDr. Premysl Zaskodny, CSc.
Emy Destinove 17, CZ-370 01 Ceske Budejovice, Czech Republic
e-mail: pzaskodny@gmail.com

Co-Authors of Individual Chapters:
Premysl Zaskodny (Czech Republic), Galina Ruban (Russian Federation, galina.ruban@seznam.cz), Lukas Fil (Czech Republic, filluk00@gmail.com), Jiri Urban (Czech Republic, hasi.urban@seznam.cz), Marketa Korandova (Czech Republic, korandova.marketa@seznam.cz)

Affiliation of Global Author:
The University of South Bohemia, Ceske Budejovice, Czech Republic
The University of Finance and Administration, Prague, Czech Republic

Reviewers:
Assoc. Prof. PhDr. Helena Zaskodna, CSc., Czech Republic
Assoc. Prof. Ing. Vladislav Pavlat, CSc., Czech Republic

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Foreword

The monograph "Educational & Didactic Communication 2014 (Vol.1: Application of Quantitative Research Degrees)" is a follow-up to earlier monographs presenting the results of scientific research methodology (see also Zaskodny, Zaskodna, 2014) in the field of science communication. The science communication is connected with a presentation of scientific research reports and can be taken as an integral part of the theory of educational communication. The theory of educational communication is based on the Doyle´s content pedagogy and the Brockmeyer´s communicative conception.

Our intention in the monograph "Educational & Didactic Communication 2014 (Vol.1: Application of Quantitative Research Degrees)" is to carry on the tradition created by monographs Educational & Didactic Communication 2010, 2011, 2012, 2013.

The monograph "Educational & Didactic Communication 2010" specified the complex and partial data mining tools in statistics, physics and psychology education (Tarabek, Zaskodny, 2011).

The monograph "Educational & Didactic Communication 2011" specified the curricular process and its association with educational data mining (Zaskodny et al., 2012).

The monograph "Educational & Didactic Communication 2012 (Vol.1: Educational Data Mining and Its Application)" dealt with the applications of educational data mining in statistics and physics (Zaskodny, 2013).

The monograph "Educational & Didactic Communication 2013 (Vol.1: Algorithms as Significant Result of data Mining Approach)" specified the algorithms role within scientific research methodology in the areas of statistics, curricular process and financial derivatives (Zaskodny, et al. (2014).

The monographs "Educational & Didactic Communication 2012, 2013, 2014" are also the result of the "1st, 2nd and 3rd International e-Conference on Optimization, Education and Data Mining in Science, Engineering and Risk Management (OEDM-SERM 2011, OEDM-SERM 2012, OEDM-SERM 2013/2014 – see www.oedm-serm.org)".

The monograph **"Educational & Didactic Communication 2014 (Vol.1:Application of Quantitative Research Degrees)"** is consisted of five parts:

Part A (The Way of Degrees Use within Scientific Research) was processed by P.Zaskodny.

Part B (Statistical Analysis of Index E-mini S&P 500) was processed by G.Ruban.

Part C (Physics Principles of Magnetic Resonance for Radiographers) was elaborated by L.Fil.

Part D (Assessment of Safeguards to Protect Civil Aviation against Acts of Unlawful Interference at International Airports) was elaborated by J.Urban.

Part E (Statistical Analysis of Euro Bund Futures and Twitter Inc.) was processed by M.Korandova.

The monograph "Educational & Didactic Communication 2014 (Vol.1: Application of Quantitative Research Degrees)" is operating not only with quantitative research degrees (see Part A, Part B, Part C, Part D, Part E) but partly also with illustration of qualitative research degrees(see Part A). The conception of this monograph represents an attempt to find the joint base of scientific research degrees (see Part A). Such joint base can be given by the detection not only of quantitative research degrees "reporting-exploration (description and also classification)-explanation-prediction" but also of cooperating qualitative research degrees "reporting-exploration (description only)-interpretation -prediction"

The monographs "Educational & Didactic Communication" are the expression of our cooperation with significant Slovakian scientist Pavol Tarabek who has co-instituted the Curriculum Studies Research Group and who has been in the course of many years the intellectual co-promoter of scientific activities, above all in the area of scientific research methodology of educational data mining. Unfortunately, our thanks to Pavol Tarabek can be pronounced only in memoriam.

Premysl Zaskodny,
Global author of monograph

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CONTENTS of Monograph Parts and Monograph Chapters

Contents of Part A:

Contents of Part B:

Contents of Part C:

Contents of Part D:

Contents of Part E:

A. THE WAY OF DEGREES USE WITHIN SCIENTIFIC RESEARCH

A.1. THE SCIENTIFIC RESEARCH

Scientific research is the job description of a scientist – scientific research is what scientists do. The subject of scientific research is the creation of knowledge based on the solution of the problems. Part of scientific research is, apart from knowledge creation, also their dissemination and use. Knowledge creation can be characterized as an explanation and interpretation of the phenomena and objects.

A.2. THE DEGREES OF SCIENTIFIC RESEARCH

Scientific research can be in the course of the solution for a specific problem generally characterized by the four degrees, which passes through (Kerlinger, 1972, Tukey, 1977):

- The first degree of "Reporting" is based on a careful and extensive data collection (previous experiences, monitoring needs, demand from the outside), which is usually focused on the emergence of ideas about what to study and what can be expected from the research. In this degree, it is not about research, but on the definition of the simultaneous state of knowledge in the field of anticipated scientific research. Identified expectations may be reflected in the formulation of the general research questions, pre-scientific and primal hypotheses, and in particular into the primary election of own research topic.

- The second degree of "Description (Exploration)" has the character of exploratory research. Exploratory research (Exploratory Data Analysis – Tukey, 1977) is a summary of the scientific methods aimed at the exploration of the data obtained in the first degree of scientific research. The basic aim is charting the area defined by the research topic to the exclusion of discovery already discovered and any previous research errors. Based on knowledge acquired by exploratory research should be particularized the specific research questions and potential hypotheses, if any, in the form of hypotheses of theoretical and operational. The aim is also to support the selection of scientific research methods and to provide a basis for further data collection. The second degree of scientific research characterizes the exploration by operations description and classification.

- The third degree "Explanation (or Interpretation)" is a natural consequence of the description (exploration). Explanatory research generates specific methods of scientific explanation in the field of application of the explanatory function of hypothesis (confirm or reject a causal relationship between the variables examined by research). Scientific explanation also generates explanatory empirical scientific methods (procedures for further data collection, the base for further data collection is formed by exploratory research results) and explanatory generally theoretical methods. Scientific interpretation generates methods of interpretation (direct observation, interview, expert investigation, analysis of documents) in the field of the application of the interpretive function of the hypothesis (the hypothesis interprets the phenomenon on the basis of inductively acquired knowledge and becomes a precursor to the emerging theory). The third degree characterizes the scientific explanation and interpretation of operations typical of quantitative research (quantitative measurable variables) and qualitative research (qualitative variables).

- The fourth degree "Prediction" is the substantial result of description (exploration) and explanation and interpretation. Prediction is also by the way to the application of science. Prediction (from the Latin prae-, before, and dicere, say) means prognosis as a claim about what happens or does not happen in the future. The term "prediction" is usually relies on estimates based on the hypotheses of explanatory or interpretive functions. In the case of quantitative research the prediction can be combined with estimates generated by constituted theory, in the case of qualitative research the prediction can be combined with estimates arising from seeds of the new theory.

The prediction ability significantly increased the authority of sciences with a mono-causality or a small number of causes. In the field of natural science, where a condition of a small number of causes have been complied, in the past, made the prediction, which were very successfully confirmed (e.g. the discovery of planet Neptune in 1846, or the curvature of spacetime in the vicinity of the Sun as a consequence of Einstein's theory of gravitation).

In the sciences, which examine multi-causality, i.e.. complex phenomena with many causes, the prediction contains a larger or smaller measure of uncertainty (e.g., prediction of the election results in the area of social sciences).

Based on the described four degrees of scientific research it is possible to deduce that quantitative research can be characterized (Zaskodny, Zaskodna, 2014, Hubik, 1999, 2006, Kerlinger, Pedhazur, 1973, Zaskodny, 2012) by the degrees of

reporting-exploration-explanation-prediction,

while a typical sequence of qualitative research is of

reporting-exploration-interpretation-prediction.

A.3. ALGORITHM OF QUANTITATIVE RESEARCH

The algorithm of qualitative research could be described before the algorithm of quantitative research, due to the complementarity (Plotnitsky, 2012) of induction and deduction (see "double movement of reflective reasoning" referred to in Zaskodny, Zaskodny, 2014). If persons interested the algorithm of qualitative research (in the form of 19 algorithmic steps) is presented in detail within work Zaskodny, Zaskodna, 2014.

The results of the qualitative research (the seeds of new theory formed by induction way and their substantiation by output hypothesis with interpretative function) could be taken as an inspiration for the start of quantitative research (the deduction essence and input-output hypotheses with explanatory function).

The algorithm of quantitative research takes the form of a sequence of 19 algorithmic steps. Individual algorithmic steps will be utilizing the work of Zaskodny, Zaskodna (2014) succinctly defined.

1 algorithmic step

The basic orientation of the projected complex research problem, creating ideas about nature and inclusion of the class of phenomena or objects (classified phenomenon or object) in the framework of the existing theories (the application in generally theoretical methods)

2 algorithmic step

Primary data collection with inductive generalizations and possible creation of provisional hypothetic deductive ideas about pre-scientific and primal hypothesis, always in relation to the examination of the predicted class of phenomena or objects (especially the application of empirical methods)

3 algorithmic step

Characteristics of the simultaneous state of the exploration of the problem solution from the perspective of selected existing theory, narrowing a complex research problem on still undescribed aspects of the theory (the application primarily of generally theoretical methods)

4 algorithmic step

The definition of partial research problems as exploring still undescribed aspects of the selected theory through the examined class of phenomena or objects of the given theory (the application of generally theoretical methods in combination with the empirical methods)

5 algorithmic step

Hypothetic deductive presumption No.1 – the formulation of the objectives of the quantitative research with regard to inductive generalization from the primary data collection (the application of generally theoretical methods in combination with the empirical methods)

6 algorithmic step

Hypothetic deductive presumption No.2 – the formulation of the general research questions with regard to inductive generalization from the primary data collection (the application of generally theoretical methods in combination with the empirical methods)

7 algorithmic step

Hypothetic deductive presumption No.3 – the definition of research variables predominantly of non-intervening - measurable nature with regard to inductive generalization from the primary data collection, the initial considerations of intervening variables such as the constructs needed for quantitative research (the application of generally theoretical methods in combination with the empirical methods)

8 algorithmic step

Hypothetic deductive presumption No.4 – characteristics of the contribution of the investigated class of phenomena or objects for the enrichment of the selected theory on the basis of anticipated research results (the application of generally theoretical methods in combination with the empirical methods). The clarification of considerations about the constructs such as intervening variables needed for quantitative research (the application of generally theoretical methods)

9 algorithmic step

By the application of quantitative explication the formulation of theoretical hypotheses can be realized (the application of generally theoretical methods in combination with the empirical methods). Quantitative explication relates to refine the concepts defining the investigated class or objects within the quantitative research and is a prerequisite for the measurement of non-intervening variables.

Quantitative explication is given by gradual specification on the basis of the transformations (see Hubik, 2006, 1999):

- of qualitative concepts (classificatory representation of the properties by the words on the basis of identification, description and subsequent relational and causal analysis) on the topological concepts (location of some properties next to the other properties on the basis of comparative analysis),
- of topological concepts on the metric concepts (representation of a topological-comparative property using the numeric relational system).
- During these transformations should be kept, for example, bijective homomorphism of projection among gradually transformed systems.

Through allowing measurability of non-intervening variables the implementation of quantitative explication is the basic premise for the formulation of theoretical hypothesis.

10 algorithmic step

Structural projection of the intervening variables (constructs), and non-intervening variables (measurable variables) into the shape of the theoretical hypothesis and formulation of operational hypothesis (the applications of generally theoretical methods)

11 algorithmic step

Projection of hypothetic deductive presumptions No.1-4 into the shape of the operating hypothesis with making her of operationalisation.

As a result of realized operationalisation should be the procedure of quantitative research, such as the order of the applied methods and techniques that will allow the secondary data collection for the verification of the operating hypothesis (the application of generally theoretical methods)

12 algorithmic step

Achieving the results No.1 by the application of quantitative research procedure – creation of data files, such as assigning the values to the non-intervening research variables (the application of empirical methods)

13 algorithmic step

Achieving the results No.2 by the application of quantitative research procedure – the application of one-dimensional analysis (the application of examination methods of one-dimensional data files, e.g. in the form of non-parametric testing – Zaskodny, Havrankova, Havranek, Vurm, 2011)

14 algorithmic step

Achieving the results No.3 by the application of quantitative research procedure – the application of two-dimensional analysis and multi-dimensional analysis with stating dependencies among the intervening and non-intervening variables, with the possible reduction of dimensionality and with the possible more general characteristics of the investigated class of phenomena and objects based on more general classification of phenomena or objects into groups.

The results No.3 can be achieved by the methods of investigation of two-dimensional and multi-dimensional data files such as in the form of regression and correlation analysis, analysis of variance, factor analysis, cluster analysis, etc.

Achieving the summarizing result No.3 in the form of confirmation or rejection of the operating hypothesis that has explanatory function (application of investigation methods of two-dimensional and multi-dimensional data files)

15 algorithmic step

Discussion of the results No.2 and No.3 – on the basis of the confirmation or rejection of operating hypothesis, the application of its explanatory function to explain the examined aspects of the class of phenomena or objects within the selected theory (the application of generally theoretical methods, on the basis of the results of the investigation methods of two-dimensional and multi-dimensional data files)

16 algorithmic step

Formulation of research conclusions No.1 – enrichment of the starting selected theory about the new aspects in the context of the examined class of phenomena or objects (the application of generally theoretical methods).

The research conclusions No.1 formulated represent the reflection of the theoretical goals of science (see Zaskodny, Zaskodna, 2014, Molnar, 2012):

"The theoretical goal of science is to build up the general laws as a knowledge system that describes the behaviour of defined groups of empirical phenomena or objects (i.e. to build a theory in the relevant scientific branch) ".

Formulated the research conclusions No.1 represent the possible theoretical contributions of quantitative research.

17 algorithmic step

Formulation of research conclusions No.2 – application of new knowledge about the starting selected theory (based on the hypothetic deductive examination of class of phenomena or objects, and on the basis of inductive generalization of contacts with reality) in terms of the possibilities of practical use (the application of a combination of empirical and generally theoretical methods).

Formulated the research conclusions No.2 represent the reflection of practicality objective of science (see Zaskodny, Zaskodna, 2014, Molnar, 2012):

"The objective of practicality is to apply the established system of knowledge on improving things, on support of progress (i.e. to meet the criterion of practicality of knowledge system built in the distant future) ".

Formulated the research conclusions No.2 represent the possible practical contributions of quantitative research.

18 algorithmic step

Formulation of research conclusions No.3 – formulation of proposals for further research possibilities arising from the looming the enriched starting selected theory and from the possible practical applications (the application of generally theoretical methods)

19 algorithmic step

Formulation of research conclusions No.4 – the proposal of the research report structure, the proposal of publishing options (respecting the needs not only to create knowledge as a subject of scientific research, but also created knowledge to spread and to use).

In general, the research report should include (Punch, 1998, 2008):

- Objectives (with a link to the general research questions)
- Generally theoretical methods (the standpoint of logical link of quantitative research parts)

- Conceptual framework of quantitative research (independent and dependent variables, specific research questions with completed empirical criterion, formulated hypotheses based on quantitative explication, explanation scheme – the explanans, explanandum – Hubik, 2006)

- Potential questionnaire as a means of indirect observation, preparation of the experiment or measurement as the other means of scientific observation (reflection of research variables measurement)

- A sample of such as selected statistical set
- Empirical methods for creating data files
- The methods of data files investigation

-The results of the quantitative research as explanation and prediction, discussion of the results

- Review of the literature, list of bibliographic citations, attachments, abstract as capturing the research

Presented algorithm of quantitative research includes 19 algorithmic steps similarly to the algorithm of qualitative research.

It is possible almost certainly to expect that when creating a specific research plan the single algorithmic steps will be pooled into groups. An example might be a grouping of hypothetic deductive presumptions No.1 to No.4, a grouping of the results No.1 to No.3, a grouping of research conclusions No.1 to No.4.

Despite the expected grouping the algorithmic steps can be classified in accordance with the degrees of quantitative research reporting-exploration-explanation-prediction in the following manner (Zaskodny. Zaskodna, 2014:

Reporting – the algorithmic steps 1-3
Exploration – the algorithmic steps 4-11
Explanation – the algorithmic steps 12-16
Prediction – the algorithmic steps 17-19

A.4. ILLUSTRATION OF SCIENTIFIC RESEARCH DEGREES

A.4.1. HISTORICAL INTRODUCTION

A.4.1.1. Wave corpuscular duality, possibility of illustration of qualitative research degrees

Louis de Broglie (1892 –1987, French physicist) became by the author of the basic contribution to the quantum theory. In dissertation he introduced in 1924 the wave properties of electrons and suggested their generalization to other matter and field particles (known as the "de Broglie hypothesis" – Warnant, 1987, Pierret, 1994, Valentini, 1992, O´Connor, Robertson, 2003).

On the basis of electrons wave properties introduction the very good possibility is discovering to illustrate, by use of historical continuities, the degrees of qualitative research. The non-classified phenomenon was examined by Louis de Broglie (the wave corpuscular duality only of electrons) and the “de Broglie hypothesis” with interpretation function has become by the output result of “de Broglie qualitative research”. This interpretation has consisted in the generalization of wave corpuscular duality to other matter and field particles.

When "de Broglie hypothesis“ was experimentally confirmed in 1927, de Broglie obtained the Nobel Prize for physics in 1929 (IAQMS, 2014).

In this way, de Broglie removed yet disturbing wave corpuscular duality of photons, with whom Albert Einstein worked within an explanation of photoelectric effect in 1905.

A.4.1.2. Mathematical formalism of quantum mechanics, possibility of illustration of quantitative research degrees

The introduction of objects wave properties (pilot-wave model) was used in 1925 by Erwin Schrödinger at the formulation of quantum mechanics using the probable de Broglie waves (Schrödinger stationary and non-stationary equations). Pilot-wave model was afterwards substituted by a mathematical formalism of quantum mechanics, this model was again listed in the life by David Bohm in 1952 (Bohmian mechanics – de Broglie-Bohm theory – Ondes et mouvements, 1926, Goldstein, 2009).

The mathematical formulation of quantum mechanics is given by mathematical formalisms, enabling precise description of quantum mechanics.

In brief, the allowable values of the energy and angular momentums (and the other physics quantities) are taken as their eigenvalues (in conjunction with their eigenfunctions) of linear operators in Hilbert space (one from abstract mathematical structures – Hilbert, 1899).

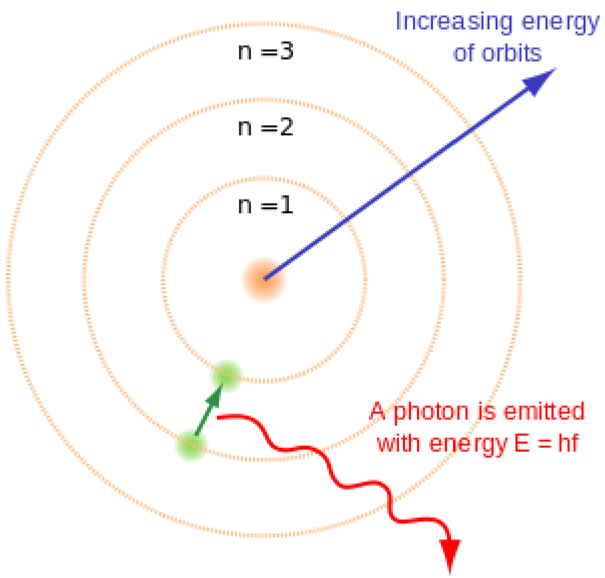
On the basis of introduction of linear operators in Hilbert space the very good possibility is discovering to illustrate, by use of historical continuities, the degrees of quantitative research. The classified phenomenon (a part of input theory as the wave corpuscular duality related to all matter and field particles) was examined by Erwin Schrödinger, Werner Heisenberg, Max Born, Pascual Jordan, John von Neumann (Neumann, 1932), Hermann Weyl and Paul Dirac (Dirac, 1925) and the input hypothesis with explanation function (the expression of physics quantities by operators) can be taken as historically supported possibility. And, on the basis of application of hypothesis explanation function, the historically presented hypothesis could be verified by means of the experimentally confirmed systems of eigenvalues and eigenfunctions connected with individual operators.

A.4.1.3. Old quantum theory

In the 1890s (Kuhn, 1978), Max Planck postulated a direct proportionality between the frequency of radiation and the quantum of energy. The proportionality constant h was later called Planck constant. In 1905, Albert Einstein explained the essence of photoelectric effect and the concept “photon” was introduced for quantum of electromagnetic energy. In 1924 (Goldstein, 2009), Louis de Broglie introduced wave corpuscular duality not only for photon, but also for electron.

The situation was changed rapidly in the years 1925–1930. The mathematical basics were postulated through the works of Erwin Schrödinger, Werner Heisenberg, Max Born, Pascual Jordan, John von Neumann (Neumann, 1932), Hermann Weyl and Paul Dirac (Dirac, 1925). The physical interpretation of the old quantum theory was also clarified by means of the uncertainty relations of Werner Heisenberg and by means of Niels Bohr complementarity principle (Plotnitsky, 2012).

The old quantum theory can be expressed through following figure:



A.4.1.4. New quantum theory

In 1930, Paul Dirac (work The Principles of Quantum Mechanics, Dirac, 1925) realized the modern abstraction of Schrödinger pilot-wave model of quantum mechanics and Heisenberg matrix model of quantum mechanics through the motions in Hilbert space.

In 1932, John von Neumann (work Mathematical Foundations of Quantum Mechanics, Neumann, 1932, 1955) deepened the classic work of Hermann Weyl. It was based on linear operators in Hilbert space (Formanek, 1983) and it was new approach in comparison with David Hilbert quadratic forms.

Although the theories of quantum mechanics continue to progress to this day (Kragh, 2002), in the 1930s, the basic framework for the mathematical formulation of quantum mechanics was realized.

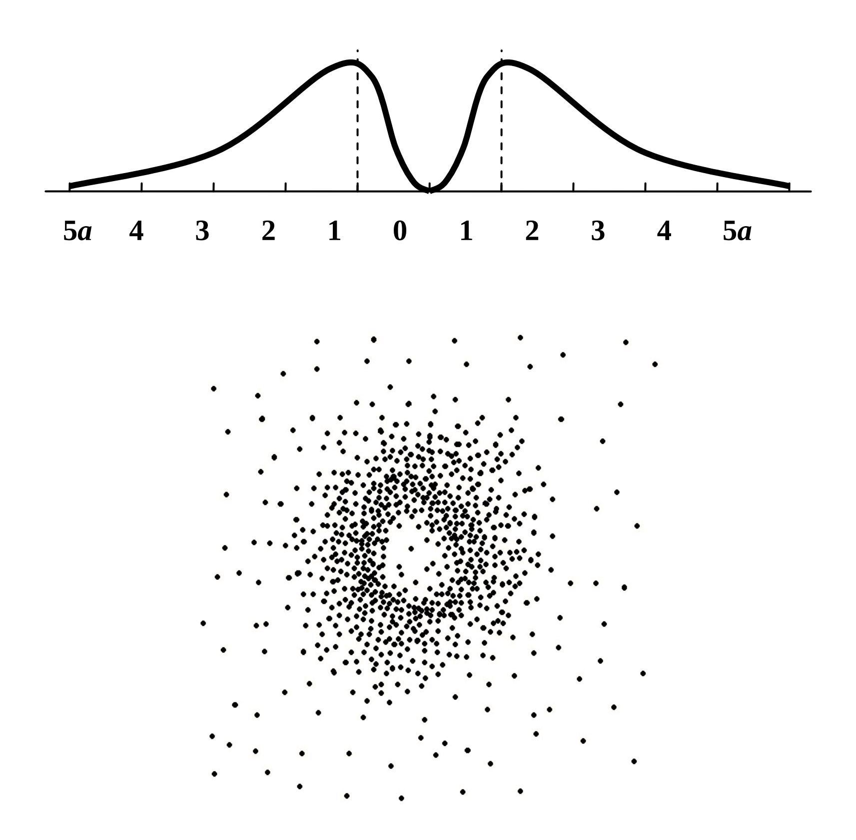
A.4.2. QUALITATIVE RESEARCH DEGREES – ILLUSTRATION OF RESEARCH OF WAVE CORPUSCULAR DUALITY THROUGH „de BROGLIE HISTORY“

A.4.2.1. The first degree – Reporting

(Excerpts from Zaskodny, 2006, 2009, Zaskodny, Prochazka, 2014 were used**)**

Graph of “probable cloud” of electron as possible result of “historical reporting”

Comparison (on the basis of new quantum theory) of “the probable cloud” with the simplest radial probability density for *n*=1, *l*=0 (*a* – Bohr radius)



Historically Described Simultaneous State (bound electron, free electron)

The electron occurs in the vicinity of the nucleus with different probabilities (see lower part of figure) - different probability distributions form different stationary states of the bound electron characterized by different shapes of “probability clouds”. The electron is prevented from escaping by the nucleus electric field. The methods of classical mechanics are unable either to find stationary states of the bound electron, or to explain e.g. the hydrogen atom stability.

The electrons came adrift from atom envelope – electrical current, Ohm´s law – the methods of classical mechanics are unable to describe the properties of the free electron.

The non-classified phenomenon (out of known theories) was identified. It could be historically connected with the beginning of qualitative research.

A.4.2.2. The second degree – Description (Exploration without Classification)

(Excerpts from Zaskodny, 2006, 2009, Zaskodny, Prochazka, 2014 were used**)**

The second degree of qualitative research is usually associated with formulation of the research questions:

How to interpret the properties of electron bound in atom envelope?

How to interpret the properties of electron within an electric current and Ohm´s law?

The formulation of research questions enables to delimit the qualitative research variables – in the case of "de Broglie approach" the wave properties and the corpuscular properties could be taken as the basic qualitative research variables.

Historically the wave properties could be given by relationships for wave length $\lambda = vT = v/f$ and Planck´s energy $E = hf$ (in another indication of frequency $E = h\nu$).

Historically the corpuscular properties could be given by relationships for mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

for momentum $p = mv$ and for Einstein´s energy $E = mc^2$.

A.4.2.3. The third degree – Interpretation

(Excerpts from Zaskodny, 2006, 2009, Zaskodny, Prochazka, 2014 were used**)**

The results based on inductive generalization should lead to incremental interpretation also to the formulation of hypotheses with an interpretative function (the seeds of new theory).

i) **Interpretation 1**

The states of electrons will be represented by the wave functions ψ which provide complete information about the state (either stationary or non-stationary). It can be expressed by the representation ®

state of electron ® by wave function ψ (state represented by wave function).

The product of the wave function ψ and its complex conjugation ψ^* presents the probability density of the electron occurrence in nucleus vicinity and the shape of "probable cloud".

The bound or free electron has got the wave properties given by de Broglie´s wave with wave length λ_{DBW} and with frequency ν_{DBW}:

$$\lambda_{DBW} = h/p = h/mv, \ \nu_{DBW} = mc^2/h.$$

The velocity of propagation of de Broglie wave v_{DBV} is greater than the velocity of light

$$v_{DBW} = \lambda_{DBW}.\nu_{DBW} = c^2/v > c.$$

ii) **Interpretation 2**

Nearly free electron in copper conductors can be seen as a particle with the specified mass and momentum. The electron (bound in the atom envelope or loose from the atom envelope) has the corpuscular properties given by mass m and momentum p

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}, \ p = mv.$$

During the passage of an electric current, for example, by copper conductor at temperature distant from absolute zero the electron is hitting on the oscillating atoms of copper – there is electrical resistance R, and the Ohm's law $U = IR$ applies for part of the electrical circuit.

iii) **Interpretation 3** – Hypothesis H1 with interpretive function

(the result 1 of qualitative research)

Based on inductive generalization the hypothesis H1 can be articulated in following form:

The electron is object of microworld characterized by wave corpuscular duality

iv) **Interpretation 4** – Hypothesis H2 with interpretive function

(the result 2 of qualitative research)

Based on deepened inductive generalization the hypothesis H2 can be articulated in following form:

All objects in the microworld (and perhaps even the world as a whole) can be characterized by the wave corpuscular duality as an essential feature of quantum dimension of physics.

A.4.2.4. The fourth degree – Prediction

(Excerpts from Zaskodny, 2006, 2009, Zaskodny, Prochazka, 2014 were used**)**

The hypotheses H1, H2 have been historically formulated as the seeds of new theory. The potential of these hypotheses with interpretive function is consisting in the possibility of a continuing quantitative research. Within such research the presented hypotheses would play the role of hypotheses with explanatory function and would be verified.

On the basis of historically described possible qualitative research the following predictions could be formulated:

i) **Prediction 1**

The wave length λ_{DBW} is negligible even for the smallest substance particles - for example, for a hydrogen molecule (m=3.3.10^{-27} kg, v =2000 m.s^{-1}, h=6.6.10^{-34} Js) it is about

$$\lambda_{DBW} = 10^{-10}\,\text{m} = 1\,\mathring{A}\,.$$

This wave length is the same size as a hydrogen atom.

ii) **Prediction 2**

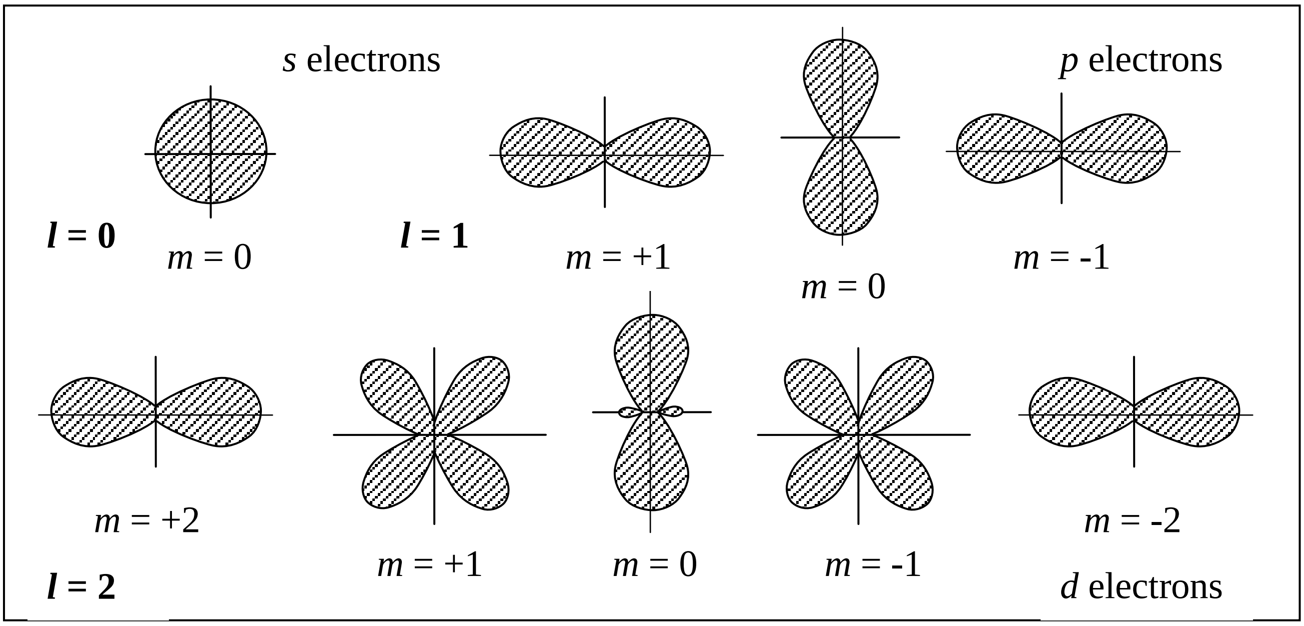
For classical electrons, it is possible to substitute from the equation ½ $m_0 v^2 = eU$ (m_0 is the electron rest mass, U is the accelerating voltage) for speed v into $\lambda_{DBW} = h/p$, and thus, to obtain the relation for the electron de Broglie wave length

$$\lambda_e = 1.23.\ 10^{-9} U^{-1/2}\ \text{m}.$$

For electrons accelerated with a voltage of 151 V, the resulting wave length is $\lambda_e = 1\,\mathring{A}$, for those accelerated with a voltage of 15 100 V the wave length is $\lambda_e = 0.1\,\mathring{A}$ (possibility of application – electron microscope).

iii) **Prediction 3**

The shapes of probabilistic clouds for the directional probabilities could be predicted from the point of view of „de Broglie history" – see following figure



Graphs of directional probability densities for *s* electrons (*l* = 0), *p* electrons (*l* = 1), and *d* electrons (*l* = 2)

iv) **Prediction 4** – Three-Dimensional Structure of Water Molecules

A simple approach could be historically applied to the prediction of the water molecule spatial structure. This approach is already issuing from the later history than "de Broglie history" – from "new quantum theory history".

The oxygen atomic envelope structure will be first examined with the aim of finding electrons that do not form pairs with parallel and antiparallel spins.

For these electrons, the shapes of their probability clouds will be analyzed based on a partially applicable analogy with the hydrogen atom.

The structure of the oxygen atomic envelope

The oxygen atomic envelope contains 8 electrons:

Within the *K orbit*, it is possible to find two electrons corresponding to probable clouds 1,0,0,1/2 and 1,0,0,-1/2.

Within the *L orbit*, it is possible to find six electrons corresponding, for example, to probable clouds

* 2,0,0,1/2 * 2,0,0,-1/2 * 2,1,-1,1/2 * 2,1,-1,-1/2 * 2,1,0,1/2 * 2,1,1,1/2.

It is obvious that only the last two electrons do not constitute the pair of electrons with parallel and antiparallel spins. These last two electrons have orbital and magnetic quantum numbers l=1, m=0 and l=1, m=1.

Based on the “standing eight” and “lying eight” of both electrons of the oxygen atom (see Prediction 3) with unpaired spins it is obvious that for the formation of a water molecule, it is necessary to have the partial “cooperation” of two hydrogen atoms that are situated in directions perpendicular one to another. If electrons of oxygen atoms have the necessary antiparallel spins, they can form homopolar bonds with the “standing eight” and “lying eight” (see Prediction 3). The angle of 104.5° (which is actually observed) can be attributed to the mutual repulsion of hydrogen atoms (for example in a similar molecule of hydrogen sulphide, this angle is 92°).

v) **Prediction 5** – Rabi method of magnetic nuclear resonance (Rabi, Ramsey, 1939)

This prediction is in the distant future (in comparison with the “de Broglie history”). Rabi method of magnetic nuclear resonance has enabled to understand the quantum and electromagnetic essence of very important imaging method in medicine – the method of magnetic nuclear resonance in radiology.

Summary of illustration of qualitative research (based on historical overtone)

The degrees sequence of qualitative research is given reporting-description (exploration without classification)-interpretation-prediction. The basic result of application of this degrees sequence is a formulation of hypothesis with interpretative function. It can be good base for quantitative research starting.

A.4.3. QUANTITATIVE RESEARCH DEGREES – ILLUSTRATION OF RESEARCH OF LINEAR OPERATORS IN HILBERT SPACE THROUGH "NEW QUANTUM THEORY HISTORY"

A.4.3.1. The first degree – Reporting

(Excerpts from Zaskodny, 2006, Zaskodny, Prochazka, 2014 were used**)**

The basic result of previous qualitative research (based on "de Broglie history") is given by two hypotheses H1 and H2 with interpretative function:

Hypothesis H1 with interpretive function

Based on inductive generalization the hypothesis H1 can be articulated in following form:

The electron is object of microworld characterized by wave corpuscular duality

Hypothesis H2 with interpretive function

Based on deepened inductive generalization the hypothesis H2 can be articulated in following form:

All objects in the microworld (and perhaps even the world as a whole) can be characterized by the wave corpuscular duality as an essential feature of quantum dimension of physics.

The hypotheses H1 and H2 are suggesting the following basic interpretation:

The wave length of de Broglie waves of a substance particle with a magnitude of the linear momentum $p=mv$ (v is the particle speed with a mass m, v being always smaller than the speed of light c) is given by the known relation

$$\lambda_{DBW} = h/p, \text{ frequency } \nu_{DBW} = mc^2/h.$$

It is easy to demonstrate that the phase speed of de Broglie waves v_{DBW} is larger than the speed of light:

$$v_{DBW} = \lambda_{DBW}.\nu_{DBW} = c^2/v > c.$$

Based on this, the concepts "probability wave", "probabilistic cloud" and their description by complex functions and numbers are substantiated. The product of a complex number with its complex conjugation presents a real number that is the squared amplitude of the de Broglie probabilistic wave - this amplitude reflects the probability that a substance particle will be found at a certain time, at a certain location.

It can be good base for quantitative research first degree "Reporting" (based on "new quantum theory history").

Within "Reporting" it is possible to take following ideas into consideration:

i) Dirac Principle of Absolute Smallness

Quantum objects (mostly micro-objects) cannot be examined without instruments. These results in serious consequences - by the very process of observation, the micro-object investigated is brought into a state different from that before the observation (for example, electrons in an atomic envelope are raised to excited states that have higher energies). Macro-objects are not endangered by this change in their state when directly observed by man - disturbances induced by the observation process are small and thus they may be ignored. In micro-objects, the disturbances cannot be ignored - the observer would otherwise acquire by means of the instrument, data on a state of the object different from the original state that was under investigation.

The Dirac principle of absolute smallness states: In nature, there is a limit of absolute smallness for observing physical objects. Above this limit, disturbances induced by the observation process may be ignored and classical mechanics may be employed; under this limit, disturbances induced by the observation process must become a part of a new theory, that of quantum mechanics.

ii) Complementarity Principle

The complementarity principle states: The principal difference between classical mechanics and quantum mechanics lies in the fact that quantum mechanics works with pairs of quantities or concepts, whose values or manifestations cannot be simultaneously determined, but both are necessary for a complete description. These pairs are referred to as "pairs of complementary quantities or concepts".

In addition to the pair "coordinate - relevant component of linear momentum", pairs of complementary quantities also include the pair "kinetic energy T - potential energy V". This means that the total electron energy expressed, for example, by the value of the Hamilton function can be determined, but it is impossible to identify what proportions correspond to the kinetic energy T and potential energy V, respectively.

The most important pair of complementary concepts describes the wave-particle duality (Plotnitsky, 2012). Physical objects have wave properties as well as corpuscular properties. When investigating them in quantum mechanics, it is possible to study either the wave characteristics (given for example by the wave length λ and frequency ν) or the corpuscular characteristics (given for example by the object mass and linear momentum). Both types of characteristics cannot be examined, but they are both necessary for the provision of complete understanding of the micro-object state and of changes in this state. For example, an electron within a probability cloud retains its corpuscular nature in its interactions with other particles, but it occurs at particular sites with different probabilities. The “probabilistic cloud” is associated with the existence of the de Broglie probabilistic waves.

iii) **Schrödinger´s formulation of quantum mechanics**

The introduction of objects wave properties (pilot-wave model) was used in 1925 by Erwin Schrödinger at the formulation of quantum mechanics using the probable de Broglie waves.

Within Schrödinger´s formulation the ideas about search of the physics quantities allowable values and allowable states have appeared. These pre-scientific and primal hypotheses have been generated by wave corpuscular duality and by old quantum theory. From here the hypothetic deductive nature of quantitative research could be depended.

The pre-scientific and primal hypotheses have been also generated by the primary experiments results and by the inductive generalization of experimentally achieved results.

A.4.3.2. The second degree – Description and Classification (Exploration)

(Excerpts from Zaskodny, 2006, Zaskodny, Prochazka, 2014 were used**)**

The wave corpuscular duality and discovered allowable values of physics quantities (e.g. Balmer´s series of spectral lines in the visible spectrum of electromagnetic radiation within hydrogen atom) have become by the examined classified phenomenon. The Dirac principle of absolute smallness has been taken into consideration. The complementarity principle has been taken into consideration.

A basic consequence of the complementarity principle is a modification of the cognitive cycle for indirectly observable micro-objects.

The use of an instrument for determining data on the micro-object leads to a sequence of the cognitive cycle:

phenomenon – experiment – mathematical model – concept – image – application.

If “direct” observation (including observation with the use of a telescope or microscope) is possible, classical mechanics usually employs the other sequence of the cognitive cycle:

phenomenon – image – concept – mathematical relation – experiment – application.

The change in the cognitive cycle could be easily demonstrated by considering the development of the concept “stationary state” in both classical and quantum mechanics (permanently the historical view “new quantum theory history” is used).

In classical mechanics, a car moving straight with a constant velocity can be directly observed; it is possible to directly acquire an image of its stationary state and to derive parameters of this state - they are given by the relation for uniform rectilinear motion $s = v.t$ and by the Newton´s first law of motion, i.e. the law of inertia.

In quantum mechanics, to describe a bound electron stationary state, it is first necessary to acquire experimental data, for example, on a spectral series (Lyman, Balmer, Paschen series, etc.) and to establish a mathematical model. Only through the mediation of this model will it be possible to obtain a probabilistic wave description (for example, by using the stationary Schrödinger equation) and to derive the distribution of the probability of finding the electron. The probability distribution is associated with the shape of the “probabilistic cloud” that can be considered as the required image of the bound electron stationary state.

The physical micro-objects investigated by quantum mechanics have been identified with the help of the Dirac principle of absolute smallness and the principle of complementarity (wave corpuscular duality as the examined classified phenomenon within historically characterized quantitative research). Their stationary states have been expressed by the “probability cloud” shape (for example, the atom in its stationary state “neither emits nor absorbs radiation”). This identification would be impossible without experiments, i.e. without implementing the process of observation with the help of a suitable instrument.

On the basis of the results achieved by the observation process, it is then necessary to describe the process of treating the results obtained with the help of a quantum mechanics mathematical model.

This mathematical model can be described by comparing the physical properties of the observation process and the mathematical characteristics of the operators. Physics started to compare these two types of properties at the moment when facts were demonstrated experimentally about the discrete nature of the values of physical quantities and thus also discrete states and the changes in these states (the Franck-Hertz experiment, the Stern-Gerlach experiment, etc.).

The results of the comparison implemented could be historically expressed by three theoretical hypotheses H1, H2, H3 in the form of interpreting postulates I1, I2, I3. The formulation of theoretical hypotheses could be taken as a consequence of the application of empirical and generally theoretical methods.

The formulation of theoretical hypotheses H1, H2, H3

The theoretical hypothesis H1 as interpreting postulate I1 states:

States of micro-objects will be represented by the wave functions ψ which provide complete information about the state (either stationary or non-stationary).

The interpreting postulate will be expressed by the representation ®

state ® by wave function ψ (state represented by wave function).

The theoretical hypothesis H2 as interpreting postulate I2 states:

Quantities A as state parameters will be represented by operators $\hat{A}$. The operators must adhere to certain conditions (for example, hermicity and linearity conditions – linear operators in Hilbert space), providing the real nature of state parameters and the validity of the general superposition principle. When examining a particular micro-object, it is necessary to delimit the so called ‘complete set of operators’ (operators from the complete set represent only simultaneously measurable quantities!) – the number of operators corresponds to the number of degrees of freedom of the problem. After implementing the so called ‘complete measurement’ (i.e. the finding of input values of quantities that are represented by operators from the complete set of operators), it is also possible to find for a given input instant the wave function of the state (i.e. state ® ψ) and to study the possible development of the state over time.

The interpreting postulate will be expressed by the representation ®

quantity A ® by operator $\hat{A}$ (quantity represented by operator).

The theoretical hypothesis H3 as interpreting postulate I3 states:

Values A_n of quantities A as parameters of particular states and functions ψ_n describing these states will be obtained by solving the so called 'eigenvalue equation' $\hat{A}\,\psi_n = A_n.\psi_n$ of the operator $\hat{A}$. The A_n obtained form a system of eigenvalues $\{A_n\}$ of the operator $\hat{A}$; the obtained functions ψ_n form a system of eigenfunctions $\{\psi_n\}$ of the operator $\hat{A}$. Indices with eigenvalues and eigenfunctions describe the fact that in the course of solving the system of eigenvalue equations of operators from the complete set of operators, individual quantum numbers will gradually appear. The set of quantum numbers corresponding to the solution of the system of eigenvalue equations will form the complete set of quantum numbers.

The interpreting postulate will be expressed by the operator $\hat{A}$ eigenvalue equation and the system of eigenvalues and eigenfunctions of the operator $\hat{A}$, i.e.

$$\hat{A}\,\psi_n = A_n\,.\psi_n\,,\ \{A_n\},\ \{\psi_n\}.$$

The end of formulation of theoretical hypotheses H1, H2, H3

The formulation of operating hypotheses H1, H2, H3, H4

Through the operationalisation of theoretical hypotheses the experiments projects could be created. Such experiments projects could to enable to measure the allowable values of physics quantities (eigenvalues) and to determine the allowable states of micro-objects (eigenfunctions). The results of experiments could be compared with the solution of eigenvalue equation of operators which have been representing the physics quantities.

Suggested mathematical and experimental procedures have become by the form of operating hypotheses, e.g. within hydrogen atom. By this way the operating hypotheses could be expressed by the form of operators which described e.g. hydrogen atom.

The hydrogen atom will be briefly described. Based on this description the formulation of operating hypotheses in the operators form will be simpler. Again, the historical view of "new quantum theory history" will be applied.

The hydrogen atom consists of a nucleus formed by a proton, and one electron in the atomic envelope. The proton mass is larger by a factor of 1836 than the electron mass - thus, the nucleus can be considered as motionless. The electron occurs in the vicinity of the nucleus with different probabilities – different probability distributions form different stationary states of the bound electron characterized by different shapes of “probability clouds”. The electron is prevented from escaping by the proton electric field. The methods of classical mechanics are unable either to find stationary states of the bound electron, or to explain the hydrogen atom stability.

The classical image of the electron would be based on two pillars from classical mechanics - Newton´s laws of motion and Coulomb´s law. Therefore, the electron would have a kinetic energy of

$$T = \frac{1}{2} mv^2,$$

Coulomb potential energy (*e* is the elementary charge)

$$V = - \frac{e^2}{4\pi \varepsilon_0 r},$$

and it would move about the proton on a circular trajectory with a radius of *r*. The third classical pillar includes the classical feature of the electromagnetic field: an electric charge that is in an accelerated motion emits energy in the form of electromagnetic radiation. This classical image results in a catastrophic prediction - the “stable” hydrogen atom would collapse in only 10^{-16} s. Due to the energy loss, the electron would approach the nucleus along a spiral trajectory till its “absorption” by the nucleus and the atom would be destroyed. The calculation of the “collapse” time is based on the total energy $E = T + V = 13.6$ eV and on the average energy emission per second.

On the basis of mentioned catastrophic prediction the research variables could be historically delimited within quantitative research degree “Exploration” – energy of electron, orbital angular momentum of electron, z-component of orbital angular momentum of electron, z-component of the spin angular momentum of electron. The operating hypotheses could be given by the form of operators which represent above mentioned research variables.

Operating hypothesis H1 – the operator of the electron energy

Operating hypothesis H2 – the operator of the squared orbital momentum of the electron

Operating hypothesis H3 – the operator of the z-component of orbital angular momentum of the electron

Operating hypothesis H4 – the matrix operator of the z-component of spin angular momentum of the electron

The complete set of operators (as the operating hypotheses H1, H2, H3, H4 from the point of view of “new quantum theory history”) comprises four operators as follows:

$E = H$ ® $\widehat{H} = \widehat{T} + \widehat{V}$	operator of the electron energy (electron Hamilton function) Operating hypothesis H1
b^2 ® $\widehat{b}^2 = -\hbar^2 \Delta_{\vartheta,\varphi}$	operator of the squared orbital angular momentum of the electron (including the Laplace operator $\Delta_{\vartheta,\varphi}$ for sphere) Operating hypothesis H2
b_z ® $\widehat{b}_z = -i\hbar \frac{\partial}{\partial\varphi}$	operator of the *z*-component of orbital angular momentum of the electron (φ is the spherical coordinate) Operating hypothesis H3
S_z ® $\widehat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	matrix operator of the z-component of the spin angular momentum of the electron Operating hypothesis H4

Complete set of operating hypotheses for the hydrogen atom

The end of formulation of operating hypotheses H1, H2, H3, H4

A.4.3.3. The third degree – Explanation

(Excerpts from Zaskodny, 2006, Zaskodny, Prochazka, 2014 were used**)**

The system of operating hypotheses H1, H2, H3, H4 with explanatory function is connected with the system of eigenvalue equations for the four operators (see theoretical hypothesis H3).

The system of eigenvalue equations of the complete set of operators will be in the following form:

$$\hat{H}\ \psi_i = E_i\ \psi_i$$
$$\hat{b}^2\ \psi_i = b_i^2\ \psi_i$$
$$\hat{b}_z\ \psi_i = b_{zi}\ \psi_i$$
$$\hat{S}_z\ \psi_i = S_{zi}\ \psi_i$$

On the left side of the eigenvalue equations, there is the operator action of particular operators on the desired eigenfunctions of the system of eigenfunctions $\{\psi_i\}$ (the system of eigenfunctions is identical for all the operators).

On the right side, there is multiplication of eigenfunctions by the appropriate eigenvalue of the system of eigenvalues of particular operators (the systems of eigenvalues are specific for each operator).

The subscript i denotes collectively all the quantum numbers occurring in the course of the solution of the system of eigenvalue equations.

The solution of the system of eigenvalue equations will be briefly reminded (again with respect to historical continuities).

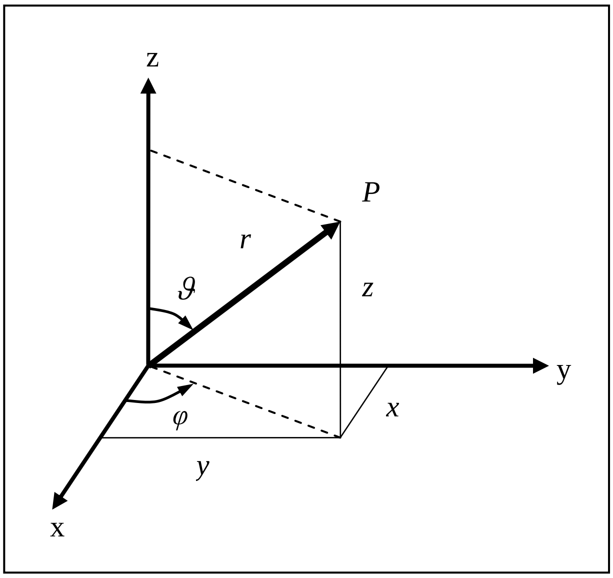
The solution of the system of eigenvalue equations (with the aim of finding a system of eigenfunctions, systems of eigenvalues and a complete set of quantum numbers) is, with respect to the central nature of the nucleus force field (a number of functions depend on the position vector magnitude r), mathematically easiest in spherical coordinates.

The Cartesian coordinates x,y,z of point P (the origin of the coordinate system will be located in the motionless atomic nucleus) will be converted to spherical coordinates r,φ,ϑ by common transformation relations (see Fig "Introduction of spherical coordinates")

$$x = r\sin\vartheta\ \cos\varphi,$$
$$y = r\sin\vartheta\ \sin\varphi,$$
$$z = r\cos\vartheta\,.$$

The angle ϑ is that between the position vector $\vec{r}$ and positive half of the z-axis, the angle φ is that between the projection of the position vector $\vec{r}$ into the coordinate plane xy and positive half of the x-axis.

Introduction of spherical coordinates



The introduction of spherical coordinates will make it possible to perform the separation of variables r, φ, ϑ in the desired eigenfunctions - the eigenfunctions will become a product of three partial functions, each of them dependent on one spherical coordinate only. The function dependent on ϑ can be expressed with the help of the so called ‘Legendre polynomials’, the function dependent on r with the help of the so called ‘Laguerre polynomials’ and the function dependent on φ can be relatively simply expressed in an exponential form. The desired eigenfunctions and the allowable eigenvalues of operators will be found by solving the eigenvalue equations corresponding to the implemented separation of variables.

In the course of solution of system of eigenvalue equations and on the basis of separation of variables r, φ, ϑ the four quantum numbers will be discovered:

The principal quantum number n
The orbital quantum number l
The magnetic quantum number m
The spin magnetic quantum number m_s

It means the systems of eigenvalues for particular operators will be closely associated with particular quantum numbers.

Their forms will be as follows:

The system of eigenvalues of the Hamilton operator $\hat{H}$: $E_n = \frac{-\,const.}{n^2}$, n is the principal quantum number taking values $n = 1, 2, \ldots$
The system of eigenvalues of the operator $\hat{b}^2$ of the squared orbital angular momentum: $b^2 = \hbar^2 l\,(l + 1)$, l is the orbital quantum number taking values $l = 0,1,\ldots, n–1$
The system of eigenvalues of the operator $\hat{b}_z$ of the z-component of orbital angular momentum: $b_z = \hbar\, m$, m is the magnetic quantum number taking values $m = -l,\ldots,0,\ldots,+l$
The system of eigenvalues of the operator $\hat{S}_z$ of the z-component of spin angular momentum: $S_z = -\hbar/2, +\hbar/2$, i.e. $S_z = m_s\hbar$, m_s is the spin magnetic quantum number taking values $m_s = -\,½, +\,½$

System of operators eigenvalues and quantum numbers for the hydrogen atom

The complete set of quantum numbers consists of the quantum numbers n, l, m, m_s. This also leads to a method of writing the system of eigenfunctions that is common to all the four operators

$$\{\psi_i\} = \{\psi_{n,l,m,m_s}\}.$$

The correspondence of eigenvalue equations solution and experiments results has showed itself to be very good. The operating hypotheses about substitution of physics quantities by operators have been confirmed not only for the hydrogen atom but also for another micro-objects. The explanatory function of operating hypotheses has been within historical continuities fully applied.

Old quantum theory (wave corpuscular duality for all micro-objects) has been not only verified but also upgraded. The improvement has been projected into the constitution of new quantum theory which has worked not only with the wave corpuscular duality but also with operation calculus (instead of the physics quantities their representations by operators in Hilbert space have been applied).

A.4.3.4. The fourth degree – Prediction (the historical chronology is not kept)

(Excerpts from Zaskodny, 2006, Zaskodny, Prochazka, 2014 were used**)**

On the basis of verified operating hypotheses H1, H2, H3, H4 has been possible to articulate several prediction generated by new quantum theory" (with respect to "new quantum theory history"), it means generated by wave corpuscular duality and operator calculus.

Prediction 1 – The non-stationary Schrödinger equation

The non-stationary Schrödinger equation describes the time development of the quantum state of a micro-object (i.e. changes in the "probabilistic cloud" shape as changes in the state). It is based on the operator $\hat{L}$ of the time change of the state. This operator is defined by

$$\hat{L} = {}^{1}\!/_{i\hbar}\,\hat{H}\,.$$

This already leads to the form of the non-stationary Schrödinger equation

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t}\,.$$

The non-stationary Schrödinger equation is frequently mentioned as an independent principle supplementing the quantum mechanics mathematical model formed by the three interpretation postulates and the superposition principle.

Prediction 2 – The stationary Schrödinger equation

The stationary Schrödinger equation is an eigenvalue equation of the Hamilton operator $\hat{H}$ – according to theoretical hypothesis H3 it can be written in the form $\hat{H}\psi_n = E_n.\psi_n$. The stationary Schrödinger equation is a direct consequence of the interpretating postulate I3 (theoretical hypothesis H3).

Under conditions delimiting the stationary state (the Hamilton operator $\hat{H}$ contains no time instruction), the stationary Schrödinger equation can be derived from the non-stationary Schrödinger equation.

Prediction 3 – Principle of Indistinguishableness, Bosons and Fermions

By defining the quantum mechanics mathematical model, assumptions are provided for investigating micro-object states and changes in micro-object states. Even one investigated micro-object itself shows its statistical nature; within its "probabilistic cloud" it is situated at different locations with different probabilities – it has its probability distribution. An even "more statistical nature" will be observed with a macrosystem of quantum micro-objects. Let us assume that the macrosystem state is represented by the wave function ψ and that "q_j"describes the *j*-th micro-object from *N* micro-objects of the macrosystem. This means that the wave function ψ can be expressed by the functional dependence

$$\psi = \psi(q_1, \ldots, q_j, \ldots, q_k, \ldots, q_N).$$

The principle of indistinguishableness states: Quantum micro-objects of one type are indistinguishable in the sense that the macrosystem state is not altered when replacing any micro-object with any other micro-object (for example, the *k*-th micro-object can be replaced with the j-th micro-object). The principle of indistinguishableness is explained by the concept of "overlapping probability clouds" of particular micro-objects.

The exchange of the *k*-th and *j*-th micro-objects is described by the particles transposition operator $\hat{P}_{jk}$. The solution of the eigenvalue equation of this operator

$$\hat{P}_{jk}\, \psi_{jk} = \lambda . \psi_{jk}$$

presents the possibility of deriving the existence of two types of indistinguishable particles – **fermions** (which adhere to Pauli´s exclusion principle, i.e. in the state defined by particular values of quantum numbers of the total set of quantum numbers, there may be at most one fermion) **and bosons** (which do not adhere to Pauli´s exclusion principle).

In the secondary application of the transposition operator $\hat{P}_{jk}$ to eigenvalue equation $\hat{P}_{jk}\, \psi_{jk} = \lambda . \psi_{jk}$, on the left side, the once exchanged micro-objects will be repeatedly exchanged – the primary action of the operator $\widehat{P}_{jk}$ will be eliminated. On the right side, after changing the order of "multiplication by the eigenvalue λ" and "action of the operator $\widehat{P}_{jk}$", the expression $\lambda^2.\psi_{jk}$ will be obtained with the help of eigenvalue equation $\widehat{P}_{jk}\,\psi_{jk} = \lambda.\psi_{jk}$. Thus, the summarized result of the primary and secondary action of the operator $\widehat{P}_{jk}$ can be written in the form

$$\psi_{jk} = \lambda^2.\psi_{jk}.$$

Therefore, there are two possible eigenvalues of the operator $\widehat{P}_{jk}$:

$$\lambda=\pm 1.$$

The value of –1 leads to the antisymmetrical wave functions (the wave function sign is changed by the transposition of two micro-objects) that are typical of fermions. The value of +1 leads to the symmetrical wave functions that are typical of bosons.

The existence of fermions and bosons is an important consequence of the indistinguishableness principle. Because fermions have "spin" that is equal to odd multiples of $\hbar/2$ and bosons have "spin" that is equal to even multiples of $\hbar/2$, it is obvious that **delimitation of the spin role is a necessary consequence of the indistinguishableness principle.**

Spin as an intrinsic angular momentum (for example of an electron) has no classical analogy; however, it is represented by the matrix operators $\widehat{S}^2$ (operator of the squared intrinsic angular momentum) and $\widehat{S}_z$ (operator of the *z*-component of the spin that has two eigenvalues $\pm\hbar/2$ characterizing the spin spatial quantization - see, for example, the "sodium doublet"). Since with the electron, there are odd multiples of $\hbar/2$, it is obvious that the electron is a fermion and that it obeys Pauli´s exclusion principle. Dirac described the spin based on relativistic mechanics, and, with the use of fourspinors, he retransformed the non-relativistic Schrödinger equation into the relativistic Dirac equation that is already in agreement with the existence of the spin.

Prediction 4 Standard model of fundamental particles and their interactions

Due to “new quantum theory history” the standard model of fundamental particles and their interactions will be taken without black mass, black energy and Higgs´s boson.

Described structures reflect the smallest known micro-objects - fundamental particles - which can be divided into the following two principal types - **fermions** (whose spin equals odd multiples of $h/4\pi$ and which obey the Pauli´s exclusion principle) and **bosons** (whose spin equals even multiples of $h/4\pi$ and which are not governed by the Pauli´s exclusion principle).

Most fundamental particles have their antiparticles with opposite charges, provided that the particle and antiparticle are annihilated in the course of their interaction. For example, in the annihilation of an electron and anti-electron (positron) both particles are annihilated producing two or three high-energy photons of the annihilation radiation. Only electrically neutral bosons are not associated with their antiparticles during their origination or annihilation.

Fermions can be divided into quarks and leptons and fermion hadrons (baryons). Quarks and leptons are formed by three generations (so called ‘flavours’) of quarks, electrons and neutrinos. Every generation comprises two quarks, an electron or some of its variants and one neutrino species (see Table).

A total of 6 types of quarks (u – quark up, d – quark down, c – quark charm, s – quark strange, t – quark top, b – quark bottom), 3 variants of electrons (common electron, muon, tauon) and 3 species of neutrinos (electron neutrino, muon neutrino, tauon neutrino) belong to quarks and leptons. All the components studied by physics as structural elements of physical objects up to the present time consist of a combination of these three generations of quarks and leptons and their antiparticles. Electrons have a negative electric charge corresponding to the magnitude of the elementary electric charge $e = 1.6.10^{-19}$ C, which was considered as indivisible till recently. Quarks also have, in addition to their electric charge, which is 1/3 or 2/3 the elementary charge, one of three possible variants of the so called ‘colour’ charge or also ‘strong’ charge. The internal structure of quarks and leptons is unknown.

The most well-known fermion hadrons (baryons) are protons (quark composition uud) and neutrons (quark composition udd). Fermion hadrons also have their internal quark structure.

	particle	mass	charge (multiples of *e*)
1st generation			
	electron	0.00054	-1
	electron neutrino	$< 10^{-8}$	0
	up - quark u	0.0047	2/3
	down - quark d	0.0074	-1/3
2nd generation	particle	mass	charge (multiples of *e*)
	muon	0.11	-1
	muon neutrino	< 0.0003	0
	charm - quark c	1.6	2/3
	strange - quark s	0.16	-1/3
3rd generation	particle	mass	charge (multiples of *e*)
	tauon	1.9	-1
	tauon neutrino	< 0.033	0
	top - quark t	189	2/3
	bottom - quark b	5.2	-1/3

Table: Three generations of quarks and leptons (mass in proton mass units)

Bosons can also be divided into bosons with unknown internal structures (gravitons, photons, gluons, intermediate or also weak gauge bosons Z and W) and boson hadrons with internal quark structures (mesons including for example the important pion with charge +*e* and quark composition $u\bar{d}$ where $\bar{d}$ is an antiquark). All factors investigated in physics up to the present time as carriers of interactions between structural elements of physical objects, either natural or artificial, are formed by bosons.

The structures of physical objects are formed **by structural elements** and **carriers of forces acting between them.** Mutual interactions make possible the existence of a physical object as a whole. Physical objects with structures of the lowest orders have fermions as structural elements and bosons are the force carriers.

There are three structures of the lowest orders. According to the type of their interactions they can be called the Fundamental Strong Interaction, the Residual Strong Interaction and the Weak Interaction. These three interactions share one characteristic - their range is very short (they can be manifested only in the smallest microphysical objects). However, if the micro-object is within the range of their action, they are many times stronger than interactions with an unlimited range. There are two types of structures formed by interactions with unlimited ranges - the Electromagnetic Interaction and the Gravitational Interaction. These interactions are prevalent in forming structures associated with macrophysical and megaphysical objects with respect to their unlimited range. Three structures formed by interactions with very short ranges and two structures formed by interactions with unlimited ranges will be described below.

The structure **"Fundamental Strong Interaction"** is responsible for the stability of nucleons - i.e. protons and neutrons. In a simplified form, elements of this structure are quarks and carriers of the fundamental strong interaction are gluons. The strong interaction is associated with the gluon field action on the so called 'colour' (or 'strong') charge of quarks.

The structure **"Residual Strong Interaction"** is responsible for the stability of atomic nuclei. Elements of this structure are in a simplified form, nucleons, and carriers of the residual strong interaction are pions (mutual interactions are, however, present between the quarks and gluon field).

The structure **"Weak Interaction"** is responsible not for the stability of micro-objects, but for their conversion or possibly decay. Structural elements of these transfiguration structures are also fermions (quarks, leptons), and carriers of the weak interaction are intermediate (weak gauge) bosons Z^0, W^- and W^+. The weak interaction is associated with effects on the so called 'weak' charge of relevant structural elements - the weak charge may be characterized with the help of the isospin. The isospin is closely related to the so called 'multiplicity', that is, a sum of particles differing from each other by their electric charge only. For example, the nucleon multiplicity equals 2; the nucleon is formed by a doublet: proton, and neutron. The transfiguration structure "weak interaction" results, for example, in the conversion of a nuclear nucleon ("neutron") into another nuclear nucleon ("proton"), an electron antineutrino and an electron, which is known as nuclear beta radiation. This process is based on the conversion of the quark d into the quark u and on the interaction between quarks mediated by the field of intermediate (weak gauge) bosons W^-.

The structure **"Electromagnetic Interaction"** is responsible not only for the stability of atoms as a whole, but also for the stability of a number of macro-objects. Its range is unlimited and thus it is manifested in both the microworld and macroworld. Structural elements are particular substance constituents of physical objects described by the quantity "electric charge" (for example, protons in atomic nuclei, and electrons in atomic envelope, but also carriers of macrocharges), and carriers of electromagnetic interactions are photons, which are closely associated with the electromagnetic field.

The structure **"Gravitational Interaction"** is responsible for the stability and development of many macro-objects as well as mega-objects. With respect to its weakness in comparison with strong and weak interactions, it is not manifested in the microworld within the range of these interactions. Elements of this structure are substance constituents of macrophysical and megaphysical objects described by the quantities "mass" and "energy"; carriers of the gravitational interaction are gravitons, which are closely associated with the gravitational field.

Whereas structures of the three lowest orders form microphysical objects (fundamental particles are substance constituents as well as force carriers), two structures formed by interactions with unlimited range can form not only microphysical objects, but also macrophysical objects and megaphysical objects. Enclosed table presents an outline of the four principal interactions serving as a basis for the description of three structures of the lowest order and two structures formed by interactions with unlimited ranges.

With the help of these five structures it is possible to understand the principles of any physical object as mutual interactions of substances and fields (see Table). "Substance" and "field" represent two of the principal forms of mass examined by physics: "substance form of mass" and "field form of mass". Physical objects are formed by structural elements (that are frequently formed by the substance form of mass) and by carriers of interactions between structural elements (that are frequently formed by the field form of mass). In more complex physical objects, the combination of substances and fields can be very complicated.

Name of interaction	Carrier of interaction	Mass
strong (fundamental, residual)	gluon	0
electromagnetic	photon	0
weak	intermediate bosons W, Z	86, 97
gravitational	graviton	0

Table An outline of principal interactions (rest mass of force carriers presented in units of proton rest mass)

Prediction 5 – Development of physical objects, Unitary and partial interactions

Due to "new quantum theory history" the development of physical objects and interactions will be taken without black mass, black energy and Higgs´s boson.

The nature of physical objects and thus also of the whole physical world (formed by the microworld, macroworld and megaworld and examined by statistical and non-statistical physics) can be understood as mutual interactions of substances and fields. Mutual interactions of substances and fields are associated with the concepts "structural element of the physical object", "carrier of interactions between structural elements", "three structures of the lowest order" and "two structures formed by interactions with unlimited ranges". The classification of fundamental particles (forming physical objects) into fermions and bosons and the outline of four partial interactions (strong interaction, weak interaction, electromagnetic interaction and gravitational interaction) also results from these concepts. The strong interaction is frequently divided into the strong fundamental interaction and strong residual interaction. Thus, the presence of five partial interactions is frequently considered.

The development of physical objects can be described with the help of the evolutionary stages of the universe up to the current stage of development, in which man tries to study the physical world. The evolutionary stages of the universe are a suitable illustrative basis for the description of a possible gradual disintegration of a solitary original general interaction into five partial interactions. The disintegration of the general interaction into partial interactions may be considered as a gradual reduction of the perfect symmetry of the universe. This reduction is manifested by a transition from one interaction, characterized by the same attributes throughout the universe, to partial interactions that differ from each other in their attributes. In the course of this gradual disintegration, particular physical objects have been produced, evolving into nature as we know it at the present time.

The currently accepted scientific theory of the origin and development of the universe is based on a sequence of three continuing evolutionary cosmological scenarios. The first evolutionary cosmological scenario may be termed "The Theory of Everything (TOE)". The basic feature of this scenario is the presence of a combined general interaction and its duration from the beginning of the expansion of the universe (popularly named the 'Big Bang') up to the time of 10^{-43}s. The remaining two evolutionary cosmological scenarios are based on alternating the standard and inflating expansion of the universe. The second scenario took from 10^{-43}s to 10^{-34}s and it is named the 'standard-inflation universe'. The third scenario has lasted from 10^{-34}s up to the present time and it is named the 'post-inflation standard universe'. The present time can be approximately characterized by the 15 milliard years that have elapsed from the beginning of the expansion. In the course of the first scenario - TOE -, the symmetry of the universe was not disturbed. In the course of the second scenario - the standard-inflation universe, there was a double disturbance of the symmetry. The gravitational interaction was separated and conditions for the separation of the strong interaction were established. In the course of the third scenario - the post-inflation standard universe, the originally general interaction was definitively disintegrated into five partial interactions evolving into the currently existing pattern of the universe.

i) Description of the first evolutionary scenario: TOE

The original general interaction - "Unitary interaction" - is associated with the first scenario: TOE. The theory of unitary interaction is named 'quantum geometrodynamics' and it is actually a quantum theory of gravitation. The currently accepted scientific theory unifying the general theory of relativity and quantum mechanics into quantum geometrodynamics is based on an assumption that the universe at the very beginning of its existence passed through a period of very extreme conditions; very high temperatures, energy, and density, and the whole mass was concentrated into a very small space. At the beginning, about 15 milliard years ago, a unique event occurred, in the course of which the mass started to expand by the action of a quantum-gravitational fluctuation (deviations from the mean zero energy value expressed, for example, by the uncertainty principle in the form $\Delta E.\Delta t \geq h$). There is no problem in establishing the location of this expansion; here will do as well as any place, since in the course of the "Big Bang", different places were concentrated in a negligible germinal space.

The initial germinal space has never been a single point, but its dimension in all directions was possibly about one Planck length. The Planck length expresses a combination of the general theory of relativity (characterized by the Newton gravitational constant κ and speed of light c) and quantum mechanics (characterized by the Planck constant $\hbar = h/2\pi$). This is a combination of the theories of the megaworld and microworld in the region of the germinal space. The theoretical approach making possible this combination is currently, in addition to the "string theory", also the "twistor theory" or the "M-theory". For example, based on the theory of strings, each fundamental particle is an expression of one of the possible modes of oscillation of a closed string, whose length is comparable with the Planck length. By investigating units of constants κ, c and $h/2\pi$ it is possible to find that the expression

$$[(h.\kappa) / (2\pi.c^3)]^{1/2}$$

has a dimension of length and its value is about $1{,}6.10^{-35}$. Thus, the Planck length about 10^{-35} m involves space-time inputs κ, c as well as quantum mechanics inputs $h/2\pi$. The Planck length is usually considered in its approximate sense, and means a length which differs from 10^{-35} m by, at most, several orders of magnitude.

The general theory of relativity itself does not explain the initial rapid expansion from negligible space. Based on its application it is possible to conclude that at the time t=0, there was a zero dimension of the universe (the universe was only a point) and temperature T, energy E, and density ρ had infinite values. This signalizes that the non-quantum theoretical model of the universe, frequently referred to as the “Big Bang”, is not able to explain the beginning of the expansion. Only a combination of the general theory of relativity and quantum mechanics makes it possible to explain the origin of the universe. Temperature, energy, and density were enormous, but not infinite. All the space dimensions (for example, in accordance with the theory of super-strings, including bosons as well as fermions, the space-time dimension equals 10) were coiled to the smallest possible size corresponding approximately to the Planck length. Thus, the possible dimension of the universe has never been smaller than a certain lower limit. There was only the general interaction - “Unitary Interaction” - and the universe was perfectly symmetrical.

ii) Description of the second evolutionary scenario of the standard-inflation universe

After the elapse of the Planck time 10^{-43}s (time from the completion of the scenario TOE) three spatial dimensions were chosen for expansion, for example, according to the string theory. The other dimensions retained their original Planck size. The temperature was about 10^{32} K (i.e. about a 10^{24} times higher temperature than that in the Sun’s interior) and the universe passed through the first reduction of its symmetry: from the general “Unitary Interaction”, the gravitational interaction was separated and in the great unification theory - GUT - the weak interaction, electromagnetic interaction and strong interaction remained combined. Under these conditions, the second, standard-inflation evolutionary scenario was implemented, lasting about 10^{-34}s from the origin of time. Within the scope of the standard-inflation evolutionary scenario, after the standard pre-inflation expansion, the inflation expansion was prevalent in the short period between 10^{-36}s and 10^{-34}s. Expansion occurred with increasing speed during the inflation expansion. In the case of the standard pre-inflation as well as standard post-inflation expansion, the expansion process occurred and occurs with decreasing speed - the braking of the pull of gravitational force prevails.

Thus, the universe inflation expansion was different from the standard expansion. The expansion speed increased exponentially; distances in the universe were extended about 10^{30} times in time interval of 10^{-36}s to 10^{-34}s. In this extremely short time after the Big Bang, the increase in the size of the universe was larger (in percents) than that in the course of the subsequent 15 milliard years. The explanation of the inflation expansion is based on consideration of a repellent “anti-gravitational” force that occurred as a consequence of the pre-inflation standard expansion and a temperature drop under the critical temperature, after which the second reduction in the symmetry of the universe should have occurred by means of the separation of the strong interaction from the weak and electromagnetic interactions. The second reduction of the symmetry did not occur in spite of this decrease in the temperature.

For the understanding of this situation, it is possible to employ an analogy with phase transitions in water. For example, during the cautious lowering of water temperature, it is possible to achieve temperatures under freezing point while water remains liquid in this “under-cooled state”. The “symmetry of liquid water”, i.e. the same characteristics at all points and in all directions, in contrast to frozen ice crystals arranged in a certain direction, is not disturbed. The existence of a distinct direction is a disturbance of the water symmetry by freezing. The unstable “under-cooled state” is richer in energy in comparison with that obtained after the phase change. A similar occurrence of energy excess in the development of the universe could possibly be explained by the action of the repellent “anti-gravitational” force, and by the origination of the inflation expansion within a time interval of 10^{-36}s to 10^{-34}s. In this way, conditions were also provided for implementing the second reduction of the symmetry. At 10^{-34}s after the Big Bang, the second evolutionary cosmological scenario was completed - the scenario of the standard-inflation universe.

Experimental data are also available supporting the existence of the inflation expansion. In 1965, with the help of an antenna that was originally designed to serve in telecommunication satellites, the electromagnetic radiation from a temperature of 2.7 K was observed. This relict radiation currently remains after the Big Bang and fills the whole universe. It can be detected regardless of the direction of the antenna and its temperature is constant. As far as quantitative characterization is concerned, each m^3 of the universe (including that of our environment) contains about 400.10^6 photons that form a never-ending sea of microwave radiation as an "afterglow" of the Big Bang.

Relict radiation, homogeneously distributed in all directions, could not be generated without inflation expansion. If only standard expansion were present, all objects could not have achieved the same temperature as a necessary condition for the origination of perfectly homogeneous relict radiation. The establishment of thermal equilibrium could be provided by a signal transferring information with at most the speed of light. In the course of the action of standard expansion only, objects would have been distanced from each other, for example, by about 30 cm for a period of 10^{-9}s from the Big Bang. However, this distance could not be achieved by a signal moving with the light speed in 10^{-9}s.

iii) Description of the third development scenario of the post-inflation standard universe

The separation of the gravitational interaction from the great unification of the other interactions is a result of the second evolutionary scenario of the "standard-inflation universe". The theory of gravitational interaction is named the 'general theory of relativity' and the theory of the great unification, is named the 'grand unification theory' (GUT). The third evolutionary scenario of "the post-inflation standard universe" does not work with the quantum aspect of gravitation, and in the first second of the post-inflation standard expansion, there is a gradual disturbance of the great unification. This is manifested by "confinement" of quarks in hadrons (i.e. separation of the fundamental strong interaction) and in further seconds by the beginning of the primary nuclear synthesis, i.e. the formation of the simplest nuclei of hydrogen and helium with a simultaneous separation of the residual strong interaction. The theory of the strong interaction (fundamental as well as residual) is named 'quantum chromodynamics'. Due to the separation of radiation from matter, the electromagnetic interaction is also separated from the weak interaction in the course of further development. The theory of the combined electromagnetic and weak interaction is named the 'Weinberg-Salam theory'; the theory of the separated electromagnetic interaction is "Maxwell electrodynamics" and the theory of the separated weak interaction is "the Fermi theory".

The third evolutionary scenario of "the post-inflation standard universe" up to the present time can be divided into four evolution eras continuing over time one after the other as follows:

The hadron era:
- the "confinement" of quarks in hadrons with isolation of the fundamental strong interaction,
- the annihilation of hadrons and antihadrons with the emergence of the so called 'baryon asymmetry' (excess of nucleons over antinucleons),
- an excess of photons and leptons.

The lepton era:
- the separation of neutrinos from other matter,
- the origination of the primary nuclear synthesis, i.e. the formation of the first atomic nuclei together with the "confinement" of nucleons in the nuclei; the residual strong interaction is separated,
- continuation of further nuclear syntheses up to the production of prime matter formed by stars and first galaxies,
- the annihilation of electrons and positrons and the coming into being of the charge neutrality in the universe with only a small excess of electrons,
- photons ionizing the originating hydrogen atoms.

The era of radiation:
- photons do not ionize hydrogen atoms any more; gaseous hydrogen and helium are produced,
- the separation of radiation from matter, since gaseous hydrogen and gaseous helium are transparent for electromagnetic radiation,
- because of the separation of radiation from matter, the electromagnetic interaction is separated from the weak interaction.

The era of matter (persisting to date)
- the formation of bulky structures in the universe, i.e. galaxies, clusters of galaxies, super-clusters of galaxies,
- gravitation attempts to combine physical objects into compact formations; the pressure attempts to equilibrate non-homogeneities in the distribution of physical objects,
- relict electromagnetic radiation as a remnant of the separation of electromagnetic radiation from matter.

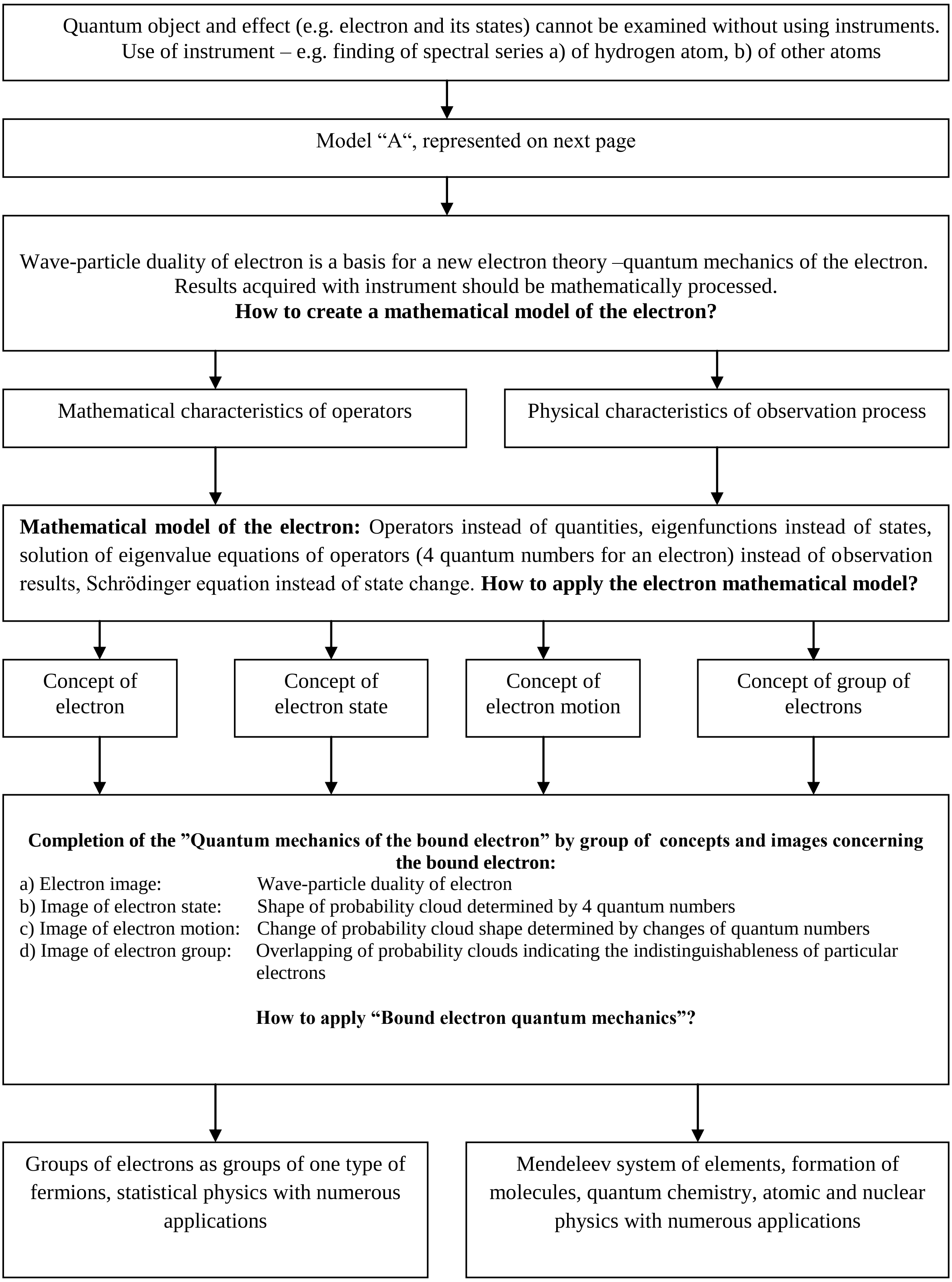
iv) The results of the development of physical objects are the physical objects existing at the present time. The accompanying result of this development is a system of five partial interactions (gravitational, electromagnetic, weak, fundamental strong and residual strong), which were gradually separated from the only general "Unitary interaction" of the pre-inflation stage.

From the point of view of illustrated degree "Prediction" it is again necessary to remind the historical continuities – due to "new quantum theory history" the development of physical objects and interactions was described without black mass, black energy and Higgs´s boson.

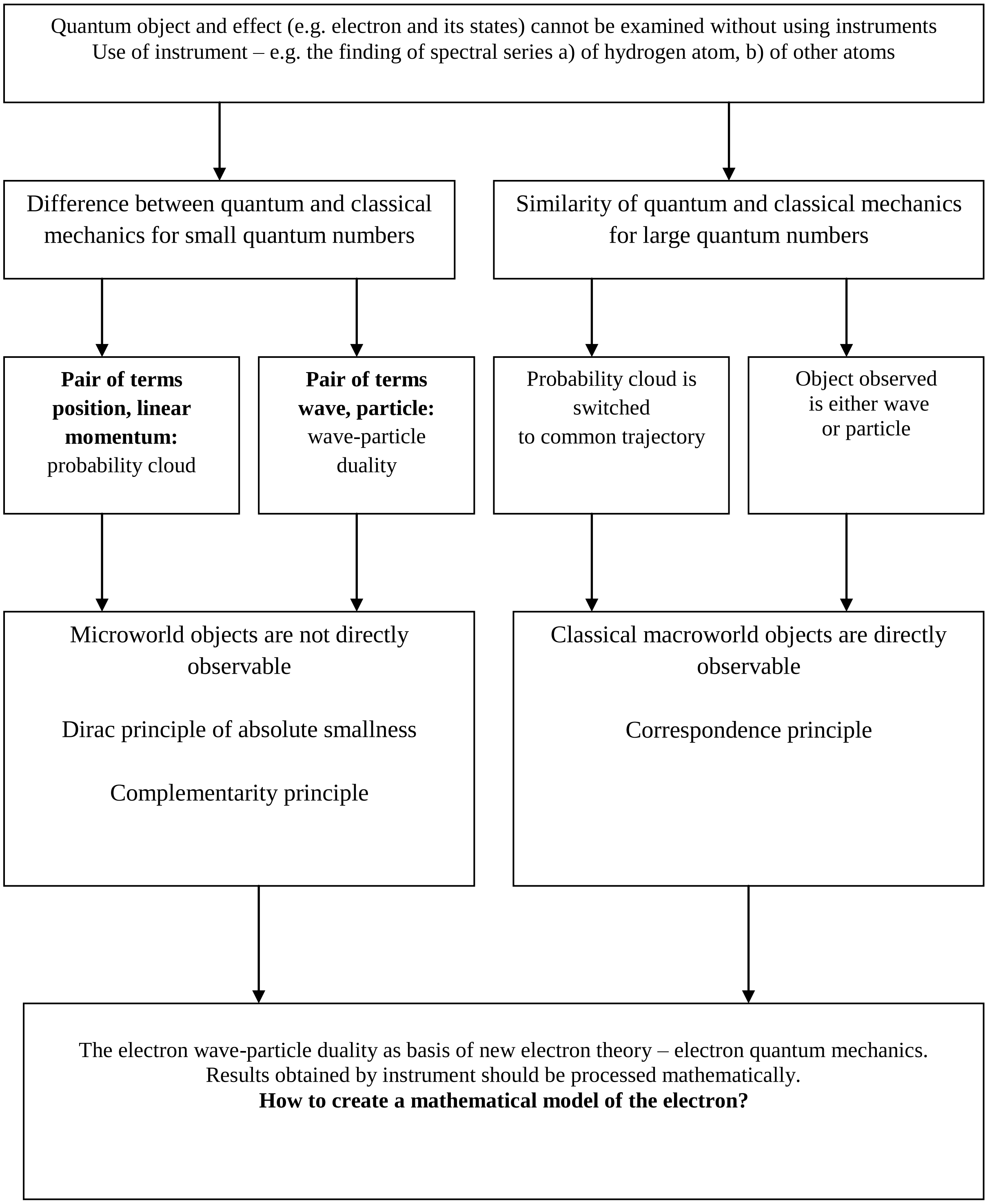
Prediction 6 – The creation of logic structure model of old and new quantum theories

The models of old quantum theory and new quantum theory can be taken as the last prediction issuing from historically described quantitative research degrees.

Representation of the Model of New Quantum Theory



Representation of the Model "A" (Model of Old Quantum Theory)



Description of the Models of Old Quantum Theory and New Quantum Theory

Objects of the microworld are not directly observable – experiments provide information about their existence. Directly observable macroworld objects (including their observation with a telescope or microscope) can be examined in a classical way as follows:

Phenomenon – Image – Concept – Mathematical relation – Experiment – Application

The quantum method is quite different:

- **a) A phenomenon** associated with the micro-object (micro-objects are not directly observable)
- **b) An experiment** (the necessary use of instruments for acquiring data on the micro-object)
- **c) A mathematical model** (processed numerical experimental values in the form of mathematical relationships)
- **d) A concept** (a concept formed without direct contact with the phenomenon examined)
- **e) An image** (an image taking advantage of classical experiences acquired by direct contact with macroworld objects)
- **f) An Application**.

This difference between the quantum method and the classical method results in requirements for establishing a new quantum theory of micro-objects - quantum mechanics. Quantum mechanics works with pairs of complementary concepts, for example, "micro-object position", "micro-object velocity". It is impossible to determine simultaneously location and velocity, in spite of the fact that both concepts are necessary for complete understanding of the micro-object principle. The basic pair of complementary concepts is "micro-object wave properties" (wave length, frequency), and "micro-object corpuscular properties" (mass, and linear momentum). Quantum mechanics investigates a new quality of physical objects that is called 'wave corpuscular duality' (old quantum theory).

This new quality resulted in abandoning physical quantities and in their representation by operators based on **primary quantization for substance particles and secondary quantization for field particles.** The operator is a mathematical instruction as to what operations should be carried out with the function situated to the right of the operator. The wave corpuscular duality results in abandoning classical motional states, and in representing quantum motional states by complex wave functions. These wave functions can be found by solving eigenvalue equations of operators as so called 'operator eigenfunctions', and it is also possible to determine simultaneously operator eigenvalues as allowable numerical values of state parameters (eigenvalues mostly have a discrete spectrum). The wave-particle duality in substance particles also leads to abandoning the classical physical object trajectory and to its replacement with the “probability cloud”. The probability cloud plays the role of a set of sites in which the micro-object examined occurs with different probabilities.

The model of the quantum non-statistical physics exemplified by the case of the electron (**i.e. in the field of primary quantization**) describes the quantum mechanics main method for investigating stationary states (the shape of the “probability cloud” remains unaltered, in contrast to the development of non-stationary states in time):

a) Physical definition of the problem and determination of quantum initial conditions,

b) Delimitation of a complete set of quantities and corresponding operators,

c) Writing and solution of a system of operator eigenvalue equations,

d) Determination of a system of eigenfunctions and system of eigenvalues of operators characterized by quantum numbers,

e) Determination of the shapes of “probability clouds”,

f) Interpretation of results based on the allowable values of quantum numbers.

A particularly important operator is the Hamilton operator (representing energy) whose eigenvalue equation is the stationary Schrödinger equation. The operator of the time change of the state (closely associated with the Hamilton operator) results in the non-stationary Schrödinger equation that makes it possible to describe the development of the quantum state over time.

The final result of the process is a set of principal concepts and images about the established substantial object of microworld, based on the application of primary quantization:

a) An image of the electron as a manifestation of the wave-particle duality,

b) An image of the electron stationary state as a shape of the “probability cloud”,

c) An image of the electron motion as that of a state change expressed by changing the shape of the “probability cloud”,

d) An image of an electron group as that of overlapping “probability clouds” associated with the indistinguishableness of electrons.

The simplest applications lead to the determination of the stationary states of bound electrons in the atomic envelope, to the determination of the spatial structure of molecules, and to quantum models of the atomic nucleus. An important part of the applications is the identification of two principal types of particles - bosons and fermions as a consequence of applying the quantum principle of indistinguishableness. From this point, it is only a small step to statistical physics and to its numerous applications.

The model of the quantum non-statistical physics is a model that is based on electron non-relativistic quantum mechanics. The switch to relativistic quantum mechanics by a simple generalization of the knowledge of non-relativistic quantum mechanics is impossible. For example, the Heisenberg uncertainty principle $\Delta x \Delta p_x \sim \hbar$ states that a simultaneous measurement of the electron coordinate and linear momentum along the x-axis is impossible (the more precise the measurement of one of these quantities, the larger the imprecision of the measurement of the second quantity), however, any of these quantities could be measured in the course of an arbitrarily short time interval separately with an arbitrarily high precision.

The existence of the limit speed (speed of light) in relativistic physics quite changes the situation. The highest achievable precision of the measurement, for example, of the linear momentum, is given by the fraction $\hbar / c$ at the given time interval Δt. In relativistic quantum mechanics it is impossible to provide an arbitrarily precise measurement of the electron linear momentum. The electron coordinate can also be measured only with a certain precision (the meaning of the “electron location” is even more restricted due to this situation).

In relativistic quantum mechanics, when considering uncertainty relations, the problem of introducing electron spin also occurs. As electron spin does not result from the solution of the Schrödinger equation, it would be necessary to achieve relativistic invariance by creating a theory of four-dimensional spinors that would lead to replacing the Schrödinger equation with the relativistic wave equation. This equation was derived for a free particle by Dirac in 1928 and thus, it was named the ‘Dirac equation’. The Schrödinger equation became a part of spinors.

From the point of view of illustrated degree “Prediction” it is again necessary to remind the historical continuities – due to “new quantum theory history” the presented prediction is presented without correct time classification.

A.5. SUMMARY OF PART A

Presented chapter "Illustration of Degrees of Scientific Research" shows the new continuities among elements of scientific research methodology. These new continuities were represented by means of the scientific research degrees.

Based on the described new continuities within scientific research methodology it is possible to deduce that quantitative research can be characterized by the degrees of

reporting-exploration-explanation-prediction,

while a typical sequence of qualitative research is of

reporting-exploration-interpretation-prediction.

These scientific research degrees are connected by the algorithm of quantitative research as the needful arrangement the described scientific methods into the sequence of algorithmic steps. The algorithm of qualitative research can be found in Zaskodny, Zaskodna, 2014.

The typical sequence of qualitative research "reporting-exploration-interpretation-prediction" was illustrated by research of wave corpuscular duality through "de Broglie history", the typical sequence of quantitative research "reporting-exploration-explanation-prediction" by research of linear operators in Hilbert space through "New quantum theory history".

Keywords
Degrees of scientific research, Qualitative and quantitative research, Hypothesis, Explanatory function, Interpretive function, Algorithm of quantitative research, Illustration of scientific research degrees

Klíčová slova
Stupně vědeckého výzkumu, Kvalitativní a kvantitativní výzkum, Hypotéza, Explanační funkce, Interpretační funkce, , Algoritmus kvantitativního výzkumu, Ilustrace stupňů vědeckého výzkumu

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B. STATISTICAL ANALYSIS OF INDEX E-mini S&P 500

BRIEF STRUCTURE OF QUANTITATIVE RESEARCHES

B.1. INTRODUCTION

B.1.1. Quantitative and qualitative research

Zaskodny, Zaskodna (2014) in the book "Methodology of Scientific Research" defines quantitative research as the confirmation of conditionally truthful statement (also known as hypothesis) by measurements.[1] It is needed to distinguish measurement from quantitative expectations. Quantitative expectations are related to the accuracy of terms, which define some objects and effects. Measurements relate to researched objects or effects directly. The quantitative explication is an assumption of measurement. Measurement of a stated object is not possible without describing exactly defined terms.[2]

Then he defines qualitative research as the creation of an interpretation (or hypothesis with an interpretative function) about a researched effect, seeing it as a seed of an arising new theory. The basis of the creation of new hypothesis (interpretations) is verifying the heuristic efficiency of techniques, which were used for gathering data. Heuristics is an experimental solving of researched problems, for which solving algorithms or exact methods are difficult to find. Therefore, the confirmation of the heuristic efficiency is providing degrees of approximation, quality of guesses, proportion of experience and intuition in case of researched qualitative variables, which are connected with the qualitative research.[3]

B.1.2. Theoretical definition of levels of quantitative research

The first level of a quantitative research, which is called Reporting, is a non-explorative level. It is connected with a concept of pre-scientific hypothesis and primal hypothesis. The basis of Reporting is gathering preparatory data and information (previous experience, monitoring of needs or demands from the outside).

The second level of a quantitative research, which is called Exploration, is an explorative level. It is linked with formulations of theoretical and operational hypothesis. Operations of description and classification are the basis of Exploration.

The third level of a quantitative research, which is called Explanation, is an explorative level. It is connected with operacionalization of an operational form of hypothesis. It is also linked with the usage of explanatory function of hypothesis.

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[1] p. 25

[2] p. 27

[3] p. 30

The fourth level of a quantitative research, which is called Prediction, is also an explorative level. It is connected, under a quantitative research, with other researched guesses and purposes, which were generated by established theories and verified hypothesis with an explanatory function.

B.1.3. Quantitative research in this chapter

I have chosen Chicago Mercantile Exchange (the Merc – CME, 2014, FP, 2014). It is an American financial and commodity derivative exchange based in Chicago. From 2008 the Merc, CBOT, NYMEX and COMEX are markets owned by the CME Group. Today, the Merc trades several types of financial instruments: interest rates, equities, currencies, and commodities. It also offers trading in alternative investments, such as weather and real estate derivatives, and has the largest options and futures contracts with open interest (number of contracts outstanding) of any futures exchange in the world. Trading is done through two methods; an open outcry format and the CME Globex electronic trading platform, which involves approximately 80 percent of market trading operations.

For statistical analysis I have chosen index E-mini S&P 500, which belongs to the Standard & Poor's Financial Services LLC (S&P). Index S&P 500 is an American stock market index based on the market capitalizations of 500 large companies with common stock listed on the NYSE or NASDAQ. The S&P 500 index components and their weightings are determined by S&P Dow Jones Indices. The E-Mini is the most popular equity index futures contract in the world. The E-Mini is preferred by relatively smaller companies and hedge funds. The original S&P contract is subsequently split 2:1, bringing it to 250 times the index. The national value of one contract is 50 times the value of the S&P 500 stock index.

B.1.4. Quantitative research 1 – Normality Testing

Reporting: The initial theory is the Normality Testing of Chicago Mercantile Exchange market. The initial data are the 50 closing prices' measurements of 50 trading days from 22nd August to 31st October 2014.

Exploration: Hypothesis H_0: the empirical distribution can be replaced by the theoretical distribution at the assumed level of significance.

Explanation: Parameter χ^2_{exp} was calculated. Parameter χ^2_{th} was taken from the Table of critical values of χ^2 test for comparison.

Prediction: Some economic problems are occurred on Chicago Mercantile Exchange market. These problems are linked with the development of Index E-mini S&P 500.

B.1.5. Quantitative research 2 – the Black-Scholes model

Reporting: The initial theory is the Black-Scholes theory, also known as the Black-Scholes model. The initial data are considered the spot and strike prices of the researched option, market volatility, annual risk free interest rate and the option's time to maturity.

Exploration: Hypothesis H_1: the price by the Black-Scholes model is determinable on Chicago Mercantile Exchange market.

Explanation: Parameter of market volatility was calculated on the basis of variation coefficient. Also the Call and Put prices of the researched option were calculated by the Black-Scholes model and then verified by parity model, also known as partial equilibrium model.

Prediction: "Greeks" were calculated. They show the sensitivity of the Call and Put options depending on changes of the initial data. Then the calculated prices were compared with the real market prices. The calculated prices are higher than the real market prices, so the Chicago Mercantile Exchange market does not work right.

B.1.6. Quantitative research 3 – the Binomial options pricing model

Reporting: The initial theory is the Binomial options pricing model. The initial data are spot and strike prices of the researched option, growth rate and rate of decrease, probability of growth.

Exploration: Hypothesis H_2: the price by the Binomial options pricing model is determinable on Chicago Mercantile Exchange market.

Explanation: Parameters of growth rate, rate of decrease and probability of growth were calculated. Also the Call and Put prices of the researched option were calculated by the Binomial options pricing model and then verified by parity model, also known as partial equilibrium model.

Prediction: The calculated prices were compared with the real market prices. The calculated prices are higher than the real market prices, so the Chicago Mercantile Exchange market does not work right.

B.2. NORMALITY TESTING

This work consists of an analysis of the price development of the derivative for the last 50 trading days. The statistical set has 50 measurements from 22th August to 31th October 2014. In this work 50 closing prices of the trading days of E-Mini S&P 500 are used.

B.2.1. Measurement scale

The quantitative metric scale (interval/differential scale) is the most suitable type of scale according to the type of measured variable. This type of scale allows determining the distance between two values of the measured quantity. It has also defined unit of measure (um, in this case it is U.S. dollar).

Variation range = max x - min x = 2009.25 – 1846.50 = 162.75 um

Width of interval h = 162.75/5 = 32.55 um (The scale has 5 intervals of length 32.55 um).

Statistical set includes 50 measurements of the E-Mini S&P 500 index in a scale from 1 to 5.

1: 1846.50 um – 1879.05 um

2: 1879.05 um – 1911.60 um

3: 1911.60 um – 1944.15 um

4: 1944.15 um – 1976.70 um

5: 1976.70 um – 2009.25 um

B.2.2. Table of frequency

Table 1: The result of working with 50 measurements

x_i	n_i	n_i/n	$\Sigma n_i/n$	$x_i n_i$	$x_i^2 n_i$	$x_i^3 n_i$	$x_i^4 n_i$
1	4	0.08	0.08	4	4	4	4
2	3	0.06	0.14	6	12	24	48
3	6	0.12	0.26	18	54	162	486
4	13	0.26	0.52	52	208	832	3328
5	24	0.48	1	120	600	3000	15000
	Σ50	Σ1		Σ200	Σ878	Σ4022	Σ18866

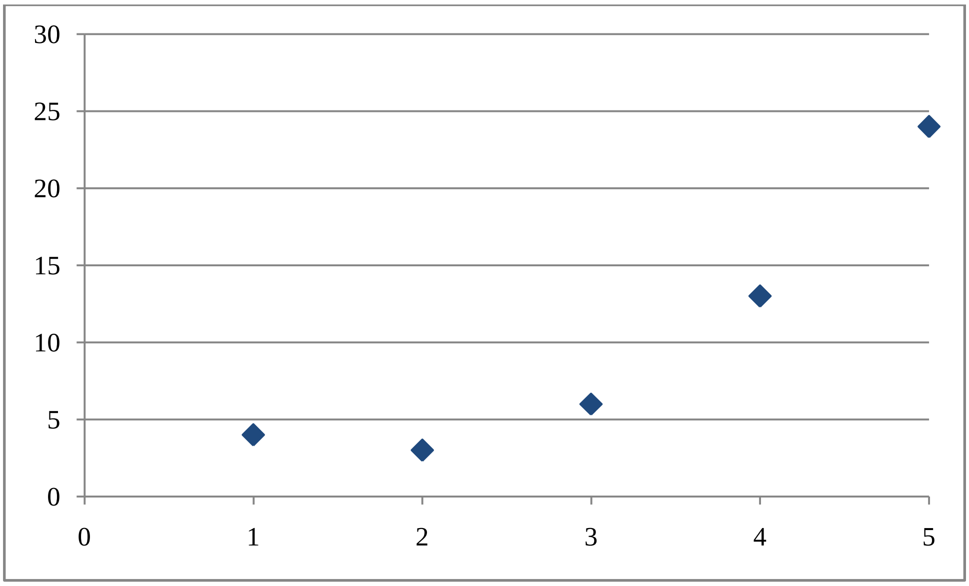
n_i = absolute frequency

$\frac{n_i}{n}$ = relative frequency

$\Sigma\frac{n_i}{n}$ = cumulative relative frequency

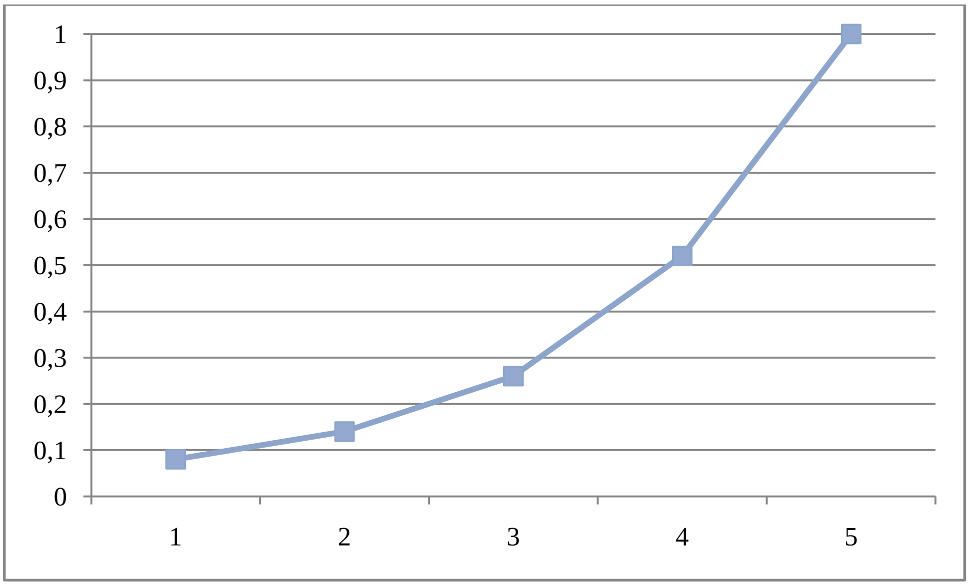
B.2.3. Empirical frequency distribution

Chart 1: Absolute frequency polygon



It can be seen from the chart of absolute frequencies that the highest frequency is in the fifth interval, which presents the highest prices of the index. This peak, or the highest result of 50 measurements, is called the mode and refers to the most frequent occurrence of the variable.

Chart 2: The chart of the distribution function, also is known as the polygon of cumulative relative frequency



B.2.4. Empirical parameters

Empirical parameters can be divided into four groups:

- position (general moment)
- variability (central moment)

- skewness (standardized moment)
- kurtosis (standardized moment)

a) *Position – the weighted arithmetic mean*

$O_r(x) = \frac{1}{n}\Sigma n_i \, . \, (x_i)^r$

$O_1(x) = 200/50 = 4$

$O_2(x) = 878/50 = 17.56$

$O_3(x) = 4022/50 = 80.44$

$O_4(x) = 18866/50 = 377.32$

b) *Variability – the emperical dispersion*

$C_2(x) = 17.56 – 4^2 = 1.56$

$C_3(x) = 80.44 – 3*17.56*4 + 2*4^3 = -2.28$

$C_4(x) = 377.32 – 4*80.44*4 + 6*17.56*4^2 -3*4^4 = 8.04$

Standard deviation: $S_x = \sqrt{1.56} = 1.249$

Standard deviation shows the explanatory power of arithmetic mean.

c) *Skewness*

$N_3(x) = \frac{-2.28}{1.56*\sqrt{1.56}} = -1.17017$

This parameter is negative, which means that the scale elements, which are on the right side from the arithmetic mean, have higher frequency.

d) *Kurtosis*

$N_4(x) = \frac{8.04}{1.56^2} = 3.303748$

$E_x = 3.303748 – 3 = 0.303748$

Excess compares kurtosis of the empirical distribution and kurtosis of the known standard normal distribution. Excess is positive, which means that the empirical distribution is sharper than standard normal distribution.

Variation coefficient: $V = \frac{S_x}{O_1} = 1.249/4 = 0.31225$

B.2.5. Testing the non-parametric hypothesis

Table 2: Results of u_i, $N(u_i)$, p_i a np_i

x_i	Interval	n_i	u_i	$N(u_i)$	p_i	np_i
1	(-∞; 1,5>	4	-2,00	0,02275	0,02275	1,1375
2	(1,5; 2,5>	3	-1,20	0,11507	0,09232	4,616
3	(2,5; 3,5>	6	-0,40	0,34458	0,22951	11,4755
4	(3,5; 4,5>	13	0,40	0,65542	0,31084	15,542
5	(4,5; ∞)	24	∞	1	0,34458	17,229

$u_i = \frac{x-\mu}{\sigma}$

$\mu = O_1 = 4$

$\sigma = S_x = 1.249$

$p_i = N(u_i) - N(u_{i-1})$

Table 3: Editing of the intervals number

x_i	n_i	np_i	$\frac{(n_i - np_i)^2}{np_i}$
1+2	7	5.7535	0.270055
3	6	11.4755	2.612618
4	13	15.542	0.415761
5	24	17.229	2.661004

$\chi^2_{exp} = \Sigma\frac{(n_i - np_i)^2}{np_i} = 5.959439$

$v = k-n-1$

k=4 (in the first and second interval, the number of observations does not have at least 5 measurement, so these two intervals are needed to be put together, that is why k=4)

$v = 4-2-1 = 1$

<u>Assumption:</u>

H_0: the empirical distribution can be replaced by the theoretical distribution

H_A: non H_0

- **Significance level: $\alpha = 0.05$**

$\chi^2_{th} = \chi^2_1\ (0.05) = 3.84$

Conclusion: $\chi^2_{th} < \chi^2_{exp}$

At the level of significance α = 0.05 the empirical distribution cannot be substitued for by the theoretical normal distribution.

- **Significance level: α = 0.025**

$\chi^2_{th} = \chi^2_1$ (0.025) = 5.02

Conclusion: $\chi^2_{th} < \chi^2_{exp}$

At the level of significance α = 0.05 the empirical distribution cannot be substituted for by the theoretical normal distribution.

- **Significance level: α = 0.01**

$\chi^2_{th} = \chi^2_1$ (0.01) = 6.63

Conclusion: $\chi^2_{th} > \chi^2_{exp}$

At the level of significance α = 0.01 the empirical distribution can be substituted for by the theoretical normal distribution.

- **Significance level: α = 0.005**

$\chi^2_{th} = \chi^2_1$ (0.005) = 7.88

Conclusion: $\chi^2_{th} > \chi^2_{exp}$

At the level of significance α = 0.01 the empirical distribution can be substituted for by the theoretical normal distribution.

Conclusion:

At the level of significance **α = 0.05 and α = 0.025** the empirical distribution **cannot** be substituted for by the theoretical normal distribution. At the level of significance **α = 0.01 and α = 0.005** the empirical distribution **can** be substituted for by the theoretical normal distribution.

B.3. BLACK-SCHOLES MODEL (a continuous model)

Table 4: Data, which are used for statistical analysis in this part of the chapter

Indicator	Value
Spot price of underlying futures contract (S)	2009.25
Strike price of underlying futures contract (X)	2100
Annual risk-free interest rate (r)	2.32%[4]
Volatility of underlying futures contract (σ)	19,83%
Time to maturity (τ)	$\frac{49}{365}$
Number of time intervals (n)	5

Strike price = 2100 um[5] (the most traded option by the volume and the open interest on 31st October 2014)

Volatility of underlying futures contract (σ):

$$\sigma = \frac{V_x}{\sqrt{\frac{1}{P}}} = 1.249*\sqrt{252} = 19.83\%$$

P = number of trading days

V_x = variation coefficient, which was calculated in the first part of the chapter.

B.3.1. Calculating suitable variables for distribution function

a) Positive variables

$$d_1 = \frac{\ln\frac{S}{X}+(r+\frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln\frac{2009.25}{2100}+(0.0232+\frac{0.1983^2}{2}).\frac{49}{365}}{0.1983\sqrt{\frac{49}{365}}} = -0.52889798$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$d_2 = -0.52889798 - 0.1983\sqrt{\frac{49}{365}} = -0.60154442$$

[4] http://www.federalreserve.gov/releases/h15/data.htm

[5] um = unit of measure (USD)

$N(d_1) = N\ (-0.53) = 0.29806 = H$ (hedge ratio)

$N(d_2) = N\ (-0.60) = 0.27425$

b) Negative variables

$(-d_1) = 0.52889798$

$(-d_2) = 0.60154442$

$N(-d_1) = N(0.53) = 0.70194$

$N(-d_2) = N(0.60) = 0.72575$

B.3.2. Calculating the right price of a Call option

$<C> = SN(d_1) – Xe^{-r\tau}N(d_2)$

$<C> = 2009.25*0.29806 - 2100*e^{-0.0232*49/365}*0.27425$

$<C> = 24.74$

The right price of a Call option is 24.74 um by the Black-Scholes model.

B.3.3. Calculating the right price of a Put option

$<P> = Xe^{-r\tau}N(-d_2) – SN(-d_1)$

$<P> = 2100*e^{-0.0232*49/365}*0.72575 – 2009.25*0.70194$

$<P> = 108.96$

The right price of a Put option is 108.96 um by the Black-Scholes model.

B.3.4. Checking by using parity model (partial equilibrium model)

$<C> + PV(X) = <P> + S$

$24.74 + 2100*e^{-0.0232*49/365} = 108.96 + 2009.25$

$2118.21 = 2118.21$

The right and the left sides of the equilibrium are the same, so the right prices of a Call and Put options were calculated correctly.

B.3.5. Greeks of a Call and a Put options

Table 5: Formulas for calculating Greeks of call and put options

Greek	Call option	Put option
Delta (Δ)	$\frac{\partial C}{\partial S}$ = N(d_1)	$\frac{\partial P}{\partial S}$ = N(d_1)-1
Gamma (Γ)	$\frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = \frac{\rho(d_1)}{S\sigma\sqrt{\tau}} = \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}S\sigma\sqrt{\tau}}$	
Vega (ν)	$\frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = S\frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}}\sqrt{\tau}$	
Theta (θ)	$-\frac{\partial C}{\partial \tau} = \frac{S\sigma e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}2\sqrt{\tau}}$ - rX$e^{-r\tau}$N(d_2)	$-\frac{\partial P}{\partial \tau} = \frac{-S\sigma e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}2\sqrt{\tau}}$ + rX$e^{-r\tau}$N(-d_2)
Rho (ρ)	$\frac{\partial C}{\partial r}$ = X$\tau e^{-r\tau}$N(d_2)	$\frac{\partial P}{\partial r}$ = -X$\tau e^{-r\tau}$N(-d_2)

Greeks shows the sensibility of a Call and Put prices on changes of the initial data (spot and strike price of the option, market volatility, annual risk free interest rate or the option's time to maturity).

Table 6: Greeks calculated for this chapter

Greek	Call option	Put option
Delta (Δ)	0.29806	-0.70194
Gamma (Γ)	0.002376	
Vega (ν)	255.36	
Theta (θ)	-201.92	-153.35
Rho (ρ)	77.08	-203.97

Delta Greek shows the change of a Call or Put option, if the spot price increases by 1 um. The Call price of the option will increase by 0.29806 um and the Put price will decrease by 0.70194 um. **Gamma Greek** shows the change of a Call or Put option between the changes of the spot price. At first, the increase by 1 um, then the decrease by 1 um are needed to be calculated. Gamma Greeks is the difference between these two measurements. The Call and Put option will increase by 0.002376 um. **Vega Greek** shows the change of a Call or Put option, if the market volatility increases by 1%. The Call and Put option will increase by 255.36 um. **Theta Greek** shows the change of a Call or Put option, if the time to maturity is a day longer. The Call price of the option will decrease by 201.92 um and the Put price will decrease by 153.35 um. **Rho Greek** shows the change of a Call or Put option, if the annual risk free interest rate increases by 1%. The Call price of the option will increase by 77.08 um and the Put price will decrease by 203.97 um.

B.4. BINOMIAL OPTIONS PRICING MODEL (a discrete model)

B.4.1.Calculating the parameters, which are needed for the binomial model

a) *the length of one period*

$$\Delta t = \frac{\tau}{n}$$

$$\Delta t = \frac{\frac{49}{365}}{5} = 0.026849$$

b) *risk-free interest rate for one period*

$$q = e^{r\Delta t}$$

$$q = 1.000623098$$

c) *up rate (growth rate)*

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$u = 1.033027$$

d) *down rate (rate of decrease)*

$$d = e^{-\sigma\sqrt{\Delta t}}$$

$$d = 0.968029$$

e) *probability of growth*

$$p = \frac{q-d}{u-d}$$

$$p = \frac{1.000623098-0.968029}{1.033027-0.968029}$$

$$p = 0.50146$$

f) *probability of decrease*

$$(1-p) = \frac{u-q}{u-d}$$

$$(1-p) = \frac{1.033027-1.000623098}{1.033027-0.968029}$$

$$(1-p) = 0.49854$$

B.4.2. Calculating the development of spot price of the underlying futures contract

$S_j = S*u^j*d^{n-j}$

$S_j^k = S*u^j*d^{k-j}$

j = the number of periods, during which the price of the underlying futures contract was increasing

(n-j) = the number of periods, during which the price of the underlying futures contract was decreasing

k = serial number of periods

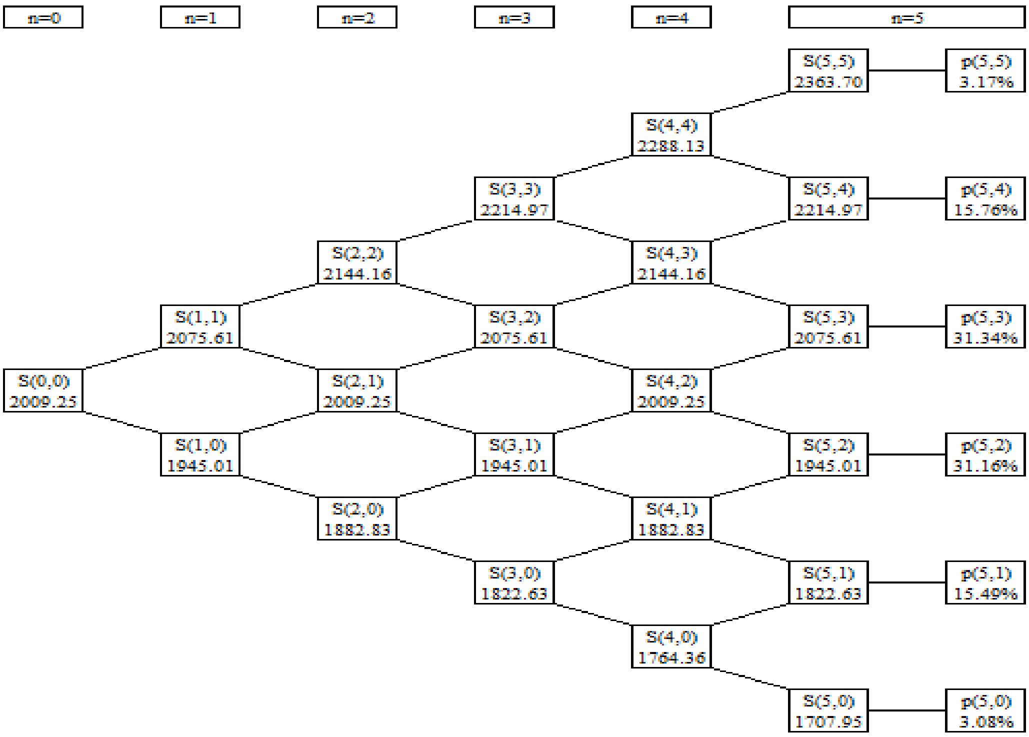
Table 7: Calculated values of the development of spot price

Spot price (a period of time and a number of growth)	Calculation	Value
S_0^0	the price, which was taken from the CME	2009.25
S_1^1	2009.25*1.033027	2075.61
S_0^1	2009.25*0.968029	1945.01
S_2^2	$2009.25*1.033027^2$	2144.16
S_1^2	2009.25*1.033027*0.968029	2009.25
S_0^2	$2009.25*0.968029^2$	1882.83
S_3^3	$2009.25*1.033027^3$	2214.97
S_2^3	$2009.25*1.033027^2*0.968029$	2075.61
S_1^3	$2009.25*1.033027*0.968029^2$	1945.01
S_0^3	$2009.25*0.968029^3$	1822.63
S_4^4	$2009.25*1.033027^4$	2288.13
S_3^4	$2009.25*1.033027^3*0.968029$	2144.16
S_2^4	$2009.25*1.033027^2*0.968029^2$	2009.25
S_1^4	$2009.25*1.033027*0.968029^3$	1882.83
S_0^4	$2009.25*0.968029^4$	1764.36
S_5^5	$2009.25*1.033027^5$	2363.70
S_4^5	$2009.25*1.033027^4*0.968029$	2214.97
S_3^5	$2009.25*1.033027^3*0.968029^2$	2075.61
S_2^5	$2009.25*1.033027^2*0.968029^3$	1945.01
S_1^5	$2009.25*1.033027*0.968029^4$	1822.63
S_0^5	$2009.25*0.968029^5$	1707.95

Table 8: Calculated values of the probability of spot price in the fifth period of time

Probability of spot price	Calculation	Value
p_5^5	$\binom{5}{5}$*0.50146^5*0.49854^0	3.17%
p_4^5	$\binom{5}{4}$*0.50146^4*0.49854^1	15.76%
p_3^5	$\binom{5}{3}$*0.50146^3*0.49854^2	31.34%
p_2^5	$\binom{5}{2}$*0.50146^2*0.49854^3	31.16%
p_1^5	$\binom{5}{1}$*0.50146^1*0.49854^4	15.49%
p_0^5	$\binom{5}{0}$*0.50146^0*0.49854^5	3.08%

Figure 1: The binomial tree for Spot price of the underlying futures contract



B.4.3. Call option

i) Calculating the right price of a Call option

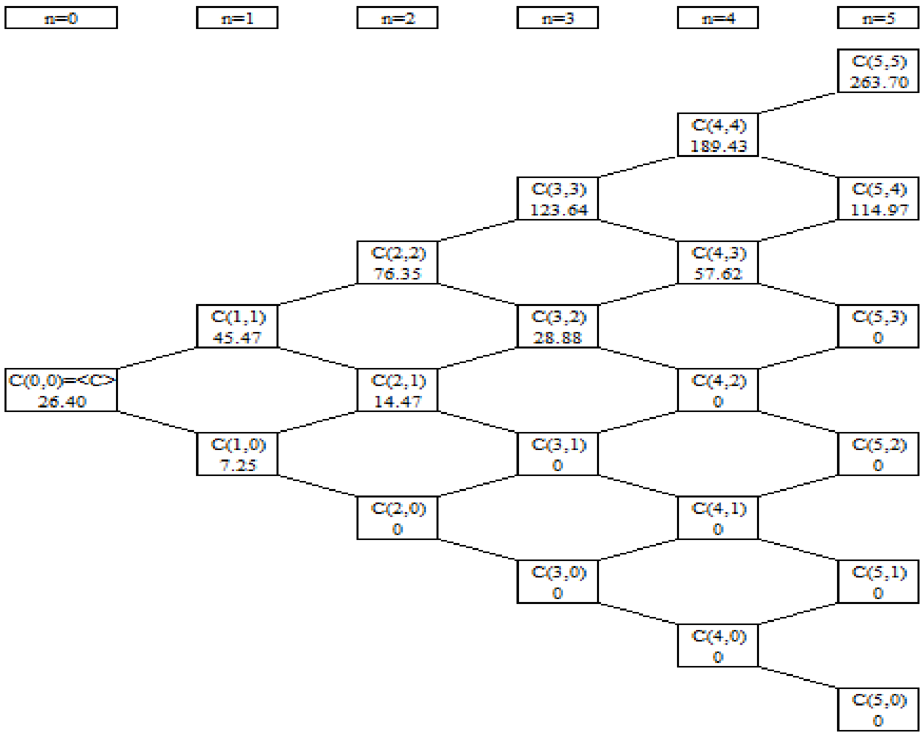
$C_j = \max(0, S_j - X)$

$$C_j^k = \frac{1}{q}(p*C_{j+1}^{k+1} + (1-p)*C_j^{k+1})$$

Table 9: Calculating the right price of a Call option

The price of a Call option	Calculation	Value
C_0^0	$\frac{1}{1.000623} * (0.50146 * 45.47 + 0.49854 * 7.25)$	26.40
C_1^1	$\frac{1}{1.000623} * (0.50146 * 76.35 + 0.49854 * 14.47)$	45.47
C_0^1	$\frac{1}{1.000623} * (0.50146 * 14.47 + 0.49854 * 0)$	7.25
C_2^2	$\frac{1}{1.000623} * (0.50146 * 123.64 + 0.49854 * 28.88)$	76.35
C_1^2	$\frac{1}{1.000623} * (0.50146 * 28.88 + 0.49854 * 0)$	14.47
C_0^2	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_3^3	$\frac{1}{1.000623} * (0.50146 * 189.43 + 0.49854 * 57.62)$	123.64
C_2^3	$\frac{1}{1.000623} * (0.50146 * 57.62 + 0.49854 * 0)$	28.88
C_1^3	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_0^3	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_4^4	$\frac{1}{1.000623} * (0.50146 * 263.70 + 0.49854 * 114.97)$	189.43
C_3^4	$\frac{1}{1.000623} * (0.50146 * 114.97 + 0.49854 * 0)$	57.62
C_2^4	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_1^4	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_0^4	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
C_5^5	S_5^5 - X = 2363.70 – 2100	263.70
C_4^5	S_4^5 - X = 2214.97 – 2100	114.97
C_3^5	S_3^5 - X = 2075.61 – 2100	0
C_2^5	S_2^5 - X = 1945.01 – 2100	0
C_1^5	S_1^5 - X = 1822.63 – 2100	0
C_0^5	S_0^5 - X = 1707.95 – 2100	0

Figure 2: The binomial tree for a Call option



The right price of a Call option by the Binomial tree model is 26.40 um.

ii) Calculating the intrinsic values of a Call option

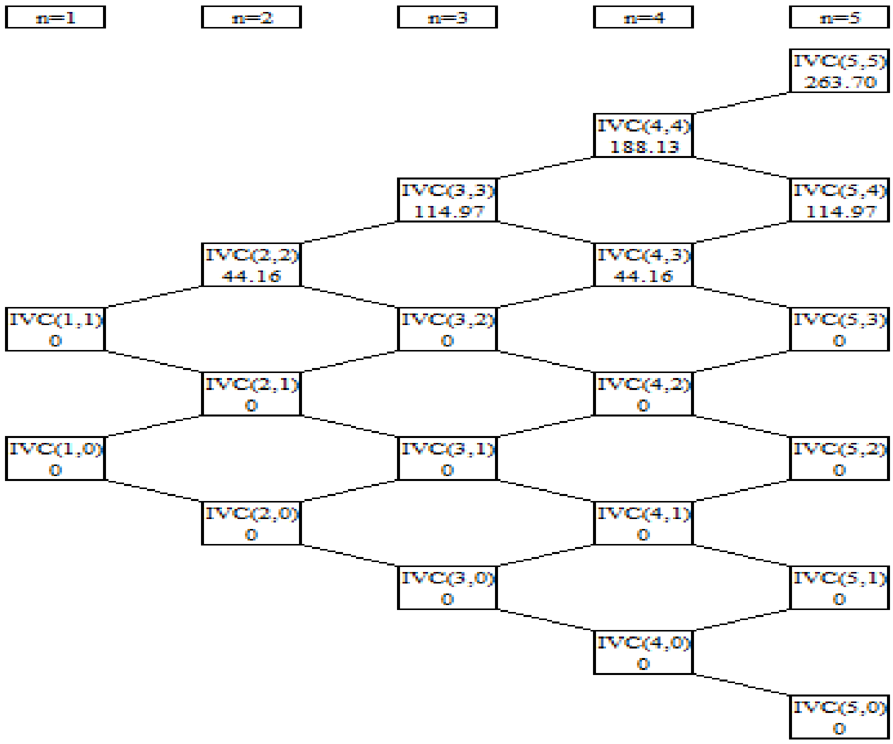
$IVC_j^k = \max(0, S_j^k - X)$

Table 10: Calculations of the intrinsic values of a Call option

The intrinsic value of a Call option	Calculation	Value
IVC_1^1	2075.61 – 2100	0
IVC_0^1	1945.01 – 2100	0
IVC_2^2	2144.16 – 2100	44.16
IVC_1^2	2009.25 – 2100	0
IVC_0^2	1882.83 – 2100	0
IVC_3^3	2214.97 – 2100	114.97
IVC_2^3	2075.61 – 2100	0
IVC_1^3	1945.01 – 2100	0
IVC_0^3	1822.63 – 2100	0
IVC_4^4	2288.13 – 2100	188.13
IVC_3^4	2144.16 – 2100	44.16
IVC_2^4	2009.25 – 2100	0

IVC_1^4	1882.83 – 2100	0
IVC_0^4	1764.36 – 2100	0
IVC_5^5	2363.70 – 2100	263.70
IVC_4^5	2214.97 – 2100	114.97
IVC_3^5	2075.61 – 2100	0
IVC_2^5	1945.01 – 2100	0
IVC_1^5	1822.63 – 2100	0
IVC_0^5	1707.95 – 2100	0

Figure 3: The binomial tree for intrinsic values of a Call option



iii) An early exercise vs. a premature sale of a Call option

Table 11: An early exercise vs. a premature sale of a Call option

An early exercise $IVC_j^k > C_j^k$	A premature sale $IVC_j^k < C_j^k$
X	$IVC_4^4 < C_4^4$
X	$IVC_3^4 < C_3^4$
X	$IVC_3^3 < C_3^3$
X	$IVC_2^3 < C_2^3$
X	$IVC_2^2 < C_2^2$
X	$IVC_1^2 < C_1^2$
X	$IVC_1^1 < C_1^1$
X	$IVC_0^1 < C_0^1$

This Call option will never have an early exercise, but there are some possibilities of a premature sale. An early exercise means that the intrinsic value of a Call option in followed period of time is higher than the price of a Call option in the same period of time. A premature sale means that the intrinsic value of a Call option in followed period of time is lower than the price of a Call option in the same period of time. A premature sale is seen in the first period of time (without depending on the number of growth or decrease), in the second period of time with at least one period of growth, in the third period of time with at least two periods of growth and in the fourth period of time with at least three periods of growth.

iv) Criterions "in the money", "at the money", "out of the money" for a Call option

In the money: $IVC_j^k > 0$

At the money: $IVC_j^k = 0$, $X = S_j^k$

Out of the money: $IVC_j^k = 0$, $X > S_j^k$

Table 12: Criterions "in the money", "at the money", "out of the money" for a Call option

In the money	At the money	Out of the money
$IVC_5^5 > 0$	X	$IVC_3^5 = 0, X > S_3^5$
$IVC_4^5 > 0$	X	$IVC_2^5 = 0, X > S_2^5$
$IVC_4^4 > 0$	X	$IVC_1^5 = 0, X > S_1^5$
$IVC_3^4 > 0$	X	$IVC_0^5 = 0, X > S_0^5$
$IVC_3^3 > 0$	X	$IVC_2^4 = 0, X > S_2^4$
$IVC_2^2 > 0$	X	$IVC_1^4 = 0, X > S_1^4$
X	X	$IVC_0^4 = 0, X > S_0^4$
X	X	$IVC_2^3 = 0, X > S_2^3$
X	X	$IVC_1^3 = 0, X > S_1^3$
X	X	$IVC_0^3 = 0, X > S_0^3$
X	X	$IVC_1^2 = 0, X > S_1^2$
X	X	$IVC_0^2 = 0, X > S_0^2$
X	X	$IVC_1^1 = 0, X > S_1^1$
X	X	$IVC_0^1 = 0, X > S_0^1$

As can be seen from the Table 12, there is not a period of time, when a Call option is "at the money", which means that the strike price of a Call option is the same as the spot price. But there are some possibilities, when a Call option is "in the money" and "out of the money". An option is "in the money" means that the strike price is lower than the calculated spot price in the followed period of time, which means that the intrinsic value is higher than zero. This variant is seen in the second period of time with two period of growth, in the third period of time with three periods of growth, in the fourth period of time with at least three periods of growth and in the fifth period of time with at least four periods of growth. An option is "out the money" means that the strike price is higher than the calculated spot price in the followed period of time, which means that the intrinsic value is zero. This variant is seen in the other periods of time.

B.4.4. Put option

i) Calculating the right price of a Put option

$P_j = \max(0, X - S_j)$

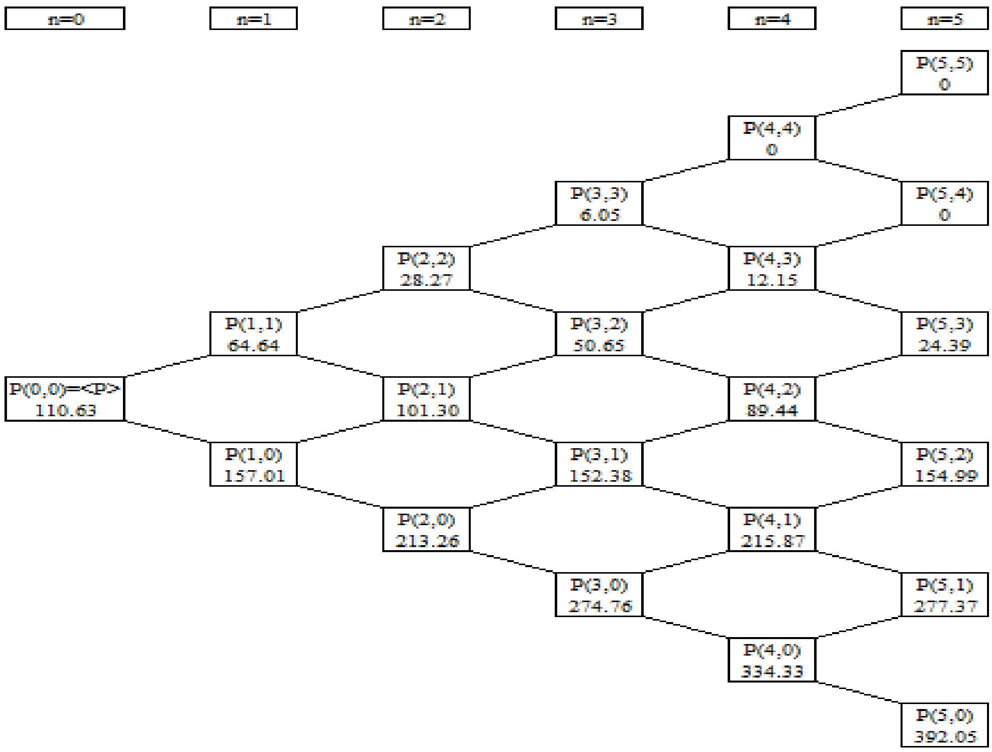
$P_j^k = \frac{1}{q}(p * P_{j+1}^{k+1} + (1-p) * P_j^{k+1})$

Table 13: Calculating the right price of a Put option

The price of a Put option	Calculation	Value
P_0^0	$\frac{1}{1.000623} * (0.50146 * 64.64 + 0.49854 * 157.02)$	110.63
P_1^1	$\frac{1}{1.000623} * (0.50146 * 28.27 + 0.49854 * 101.30)$	64.64
P_0^1	$\frac{1}{1.000623} * (0.50146 * 101.30 + 0.49854 * 213.26)$	157.02
P_2^2	$\frac{1}{1.000623} * (0.50146 * 6.05 + 0.49854 * 50.65)$	28.27
P_1^2	$\frac{1}{1.000623} * (0.50146 * 50.65 + 0.49854 * 152.38)$	101.30
P_0^2	$\frac{1}{1.000623} * (0.50146 * 152.38 + 0.49854 * 274.76)$	213.26
P_3^3	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 12.15)$	6.05
P_2^3	$\frac{1}{1.000623} * (0.50146 * 12.15 + 0.49854 * 89.44)$	50.65

P_1^3	$\frac{1}{1.000623} * (0.50146 * 89.44 + 0.49854 * 215.87)$	152.38
P_0^3	$\frac{1}{1.000623} * (0.50146 * 215.87 + 0.49854 * 334.33)$	274.76
P_4^4	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 0)$	0
P_3^4	$\frac{1}{1.000623} * (0.50146 * 0 + 0.49854 * 24.39)$	12.15
P_2^4	$\frac{1}{1.000623} * (0.50146 * 24.39 + 0.49854 * 154.99)$	89.44
P_1^4	$\frac{1}{1.000623} * (0.50146 * 154.99 + 0.49854 * 277.37)$	215.87
P_0^4	$\frac{1}{1.000623} * (0.50146 * 277.37 + 0.49854 * 392.05)$	334.33
P_5^5	X - S_5^5 = 2100 – 2363.70	0
P_4^5	X - S_4^5 = 2100 – 2214.97	0
P_3^5	X - S_3^5 = 2100 – 2075.61	24.39
P_2^5	X - S_2^5 = 2100 – 1945.01	154.99
P_1^5	X - S_1^5 = 2100 – 1822.63	277.37
P_0^5	X - S_0^5 = 2100 – 1707.95	392.05

Figure 4: The binomial tree for a Put option



The right price of a Put option by the Binomial tree model is 110.63 um.

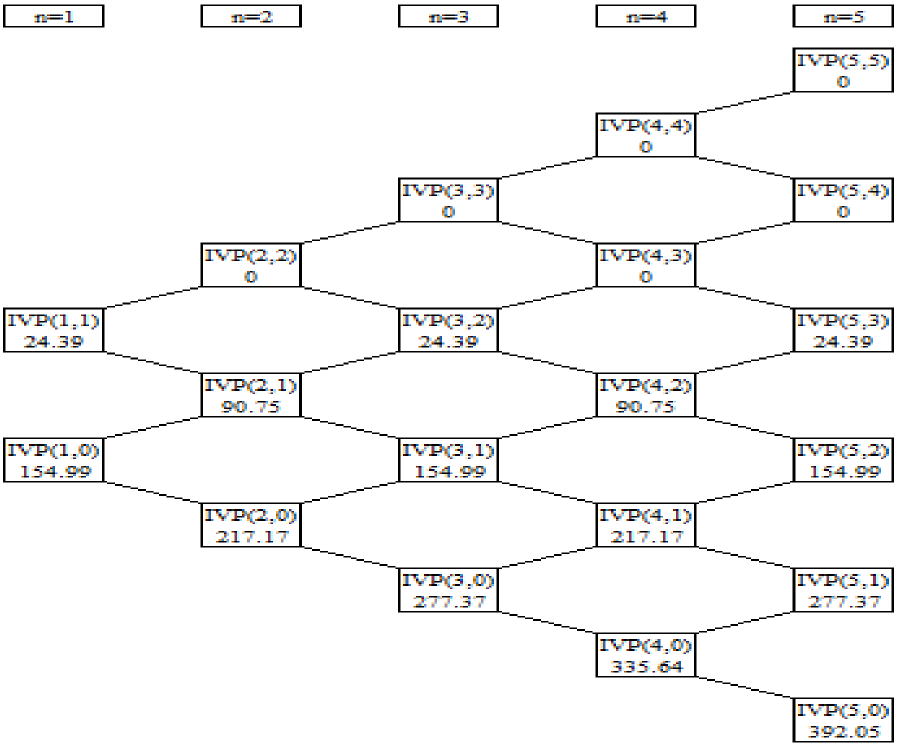
ii) Calculating the intrinsic values of a Put option

IVP_j^k = max (0, X - S_j^k)

Table 14: Calculations of the intrinsic values of a Put option

The intrinsic value of a Put option	Calculation	Value
IVP_1^1	2100 – 2075.61	24.39
IVP_0^1	2100 – 1945.01	154.99
IVP_2^2	2100 – 2144.16	0
IVP_1^2	2100 – 2009.25	90.75
IVP_0^2	2100 – 1882.83	217.17
IVP_3^3	2100 – 2214.97	0
IVP_2^3	2100 – 2075.61	24.39
IVP_1^3	2100 – 1945.01	154.99
IVP_0^3	2100 – 1822.63	277.37
IVP_4^4	2100 – 2288.13	0
IVP_3^4	2100 – 2144.16	0
IVP_2^4	2100 – 2009.25	90.75
IVP_1^4	2100 – 1882.83	217.17
IVP_0^4	2100 – 1764.36	335.64
IVP_5^5	2100 – 2363.70	0
IVP_4^5	2100 – 2214.97	0
IVP_3^5	2100 – 2075.61	24.39
IVP_2^5	2100 – 1945.01	154.99
IVP_1^5	2100 – 1822.63	277.37
IVP_0^5	2100 – 1707.95	392.05

Figure 5: The binomial tree for intrinsic values of a Put option



iii) An early exercise vs. a premature sale of a Put option

Table 15: An early exercise vs. a premature sale of a Put option

An early exercise $IVP_j^k > P_j^k$	A premature sale $IVP_j^k < P_j^k$
$IVP_2^4 > P_2^4$	$IVP_3^4 < P_3^4$
$IVP_1^4 > P_1^4$	$IVP_3^3 < P_3^3$
$IVP_0^4 > P_0^4$	$IVP_2^3 < P_2^3$
$IVP_1^3 > P_1^3$	$IVP_2^2 < P_2^2$
$IVP_0^3 > P_0^3$	$IVP_1^2 < P_1^2$
$IVP_0^2 > P_0^2$	$IVP_1^1 < P_1^1$
X	$IVP_0^1 < P_0^1$

This Put option has some possibilities of an early exercise and a premature sale. An early exercise means that the intrinsic value of a Put option in followed period of time is higher than the price of a Put option in the same period of time. It can be seen that there is a possibility of an early exercise in the second period of time without growth, in the third period of time with maximum of one growth and in the fourth period of time with maximum of two growths. A premature sale means that the intrinsic value of a Put option in followed period of time is lower than the price of a Put option in the same period of time. A premature sale is seen in the first period of time (without depending on the number of growth or decrease), in the second period of time with at least one period of growth, in the third period of time with at least two periods of growth and in the fourth period of time with at least three periods of growth.

iv) Criterions “in the money”, “at the money”, “out of the money” for a Put option

In the money: $IVP_j^k > 0$

At the money: $IVP_j^k = 0,\ X = S_j^k$

Out of the money: $IVP_j^k = 0,\ X < S_j^k$

Table 16: Criterions “in the money”, “at the money”, “out of the money” for a Put option

In the money	At the money	Out of the money
$IVP_3^5 > 0$	X	$IVP_5^5 = 0,\ X < S_5^5$
$IVP_2^5 > 0$	X	$IVP_4^5 = 0,\ X < S_4^5$
$IVP_1^5 > 0$	X	$IVP_4^4 = 0,\ X < S_4^4$
$IVP_0^5 > 0$	X	$IVP_3^4 = 0,\ X < S_3^4$
$IVP_2^4 > 0$	X	$IVP_3^3 = 0,\ X < S_3^3$
$IVP_1^4 > 0$	X	$IVP_2^2 = 0,\ X < S_2^2$
$IVP_0^4 > 0$	X	X
$IVP_2^3 > 0$	X	X
$IVP_1^3 > 0$	X	X
$IVP_0^3 > 0$	X	X
$IVP_1^2 > 0$	X	X
$IVP_0^2 > 0$	X	X
$IVP_1^1 > 0$	X	X
$IVP_0^1 > 0$	X	X

As can be seen from the Table 16, there is not a period of time, when a Put option is “at the money”, which means that the strike price of a Put option is the same as the spot price. But there are some possibilities, when a Put option is “in the money” and “out of the money”. An option is “out the money” means that the strike price is lower than the calculated spot price in the followed period of time, which means that the intrinsic value is zero. This variant is seen in the second period of time with two period of growth, in the third period of time with three periods of growth, in the fourth period of time with at least three periods of growth and in the fifth period of time with at least four periods of growth. An option is "in the money" means that the strike price is higher than the calculated spot price in the followed period of time, which means that the intrinsic value is higher than zero. This variant is seen in the other periods of time.

B.4.5. Checking by using parity model (partial equilibrium model)

$$<C> + PV(X) = <P> + S$$

$$26.40 + 2100*e^{-0.0232*49/365} = 110.63 + 2009.25$$

$$2119.88 = 2119.88$$

The right and the left sides of the equilibrium are the same, so the right prices of a Call and Put options were calculated correctly.

B.5. COMPARISON OF THEORETICAL AND MARKET PRICES

During these two parts of the chapter the right prices of call and put options were calculated. The right price of a call option is 24.74 um and the right price of a put option is 108.96 um by the Black-Scholes model (a continuous model). The right price of a call option is 26.40 um and the right price of a put option is 110.63 um in the binomial options pricing model (discrete model). These two models are considered to be very simple pricing techniques. As it can be seen, the results produced by the Black-Scholes model are a little bit lower than the result produced by the binomial options pricing model.

The real price of this option on the Chicago Mercantile Exchange was 5.50 um for a Call option and 94.00 um for a Put option. It can be seen that the real prices are lower than the right prices. It means that the market do not trade with the correct prices.

B.6. SUMMARY OF PART B

At the first part of this chapter, which tests normality of Chicago Mercantile Exchange market, were achieved the following results. At the level of significance **α = 0.05 and α = 0.025** the empirical distribution **cannot** be substituted for by the theoretical normal distribution. At the level of significance **α = 0.01 and α = 0.005** the empirical distribution **can** be substituted for by the theoretical normal distribution.

During the following two parts of the chapter the right prices of call and put options were calculated.

At the second part of this chapter, in which was used the Black-Scholes model (continuous model), were calculated the right price of a call option is 24.74 um and the right price of a put option is 108.96 um. The real price of this option on the Chicago Mercantile Exchange was 5.50 um for a Call option and 94.00 um for a Put option. It can be seen that the real prices are lower than the right prices. It means that the market do not trade with the correct prices.

At the second part of this chapter, in which was used the Binomial options pricing model (a discrete model), were calculated the right price of a call option is 26.40 um and the right price of a put option is 110.63 um. The real price of this option on the Chicago Mercantile Exchange was 5.50 um for a Call option and 94.00 um for a Put option. It can be seen that the real prices are lower than the right prices. It means that the market do not trade with the correct prices.

These two models are considered to be very simple pricing techniques. As it can be seen, the results produced by the Black-Scholes model are a little bit lower than the result produced by the Binomial options pricing model.

The presented quantitative researches are issuing not only from quantitative research degrees "reporting-exploration-explanation-prediction" (Zaskodny, Zaskodna, 2014) but also from the following sources: Board of Governors of the Federal Reserve System BGFRS (2014), Financial Portal FP (2014), Zaskodny, Pavlat, Budik (2007).

Key words

Scientific research degrees, Binomial options pricing model, binomial tree, Black-Scholes model, call option, Chi-square, index E-mini S&P 500, central moment, distribution function, general moment, Greeks, put option, random variable, standardized moment, theoretical normal distribution.

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C. PHYSICS PRINCIPLES OF MAGNETIC RESONANCE FOR RADIOGRAPHERS

C.1. THE DEGREES OF QUANTITATIVE RESEARCH

The research "Physics Principles of Magnetic Resonance for Radiographers" can be taken as typical quantitative research.

Quantitative research can be described by means of four degrees characterizing the solution of a specific problem (Záškodný, Záškodná, 2014). These four degrees will be briefly presented.

- The first degree "Reporting" is connected with data collection about the simultaneous state of anticipated scientific research. This degree is not about research.
- The second degree "Exploration (Description with Classification)" has the character of exploratory research. Within quantitative research this degree is connected with pre-scientific and primal hypotheses and also with operations "description" and "classification". The classified phenomenon (and input theory associated with the classified phenomenon) is usually creating the formulation of problem solved by a quantitative research.
- The third degree "Explanation" is issuing from the description and classification. In the course of this quantitative research degree the theoretical hypotheses are established and based on theoretical hypotheses the operating hypotheses are formed. Operating hypotheses are associated with explanatory function. The operating hypotheses enable the process of operationalization with exact formulation of research variables and with exact delimitation of research variables measurement. The result of this degree is mainly given by hypotheses verification.
- The fourth degree "Prediction" is the consequence of exploration) and explanation. The predictions are using the explanatory function of operating hypotheses after the verification of hypotheses. The predictions enable to form the possible prognoses.

On the basis of the described four degrees (creating the structure of presented research) of quantitative research it is possible to characterize the quantitative research by the sequence of degrees

reporting-exploration-explanation-prediction.

C.2. INTRODUCTION TO PRESENTED QUANTITATIVE RESEARCH

Firstly, I would like to mention that the chapter "Physics Principles of Magnetic Resonance for Radiographers[1]" is based on my bachelor's dissertation.

Currently, it is not available any simple educational text with the subject "Physics Principles of Magnetic Resonance for Radiographers" which would be adequate to knowledge of students of the study branch of Radiographer and of other related branches of study and which would be based on operational calculus.

On the grounds of this finding several partial objectives were defined which this thesis should comply with:

1) Selection of a theory of knowledge transfer to the level of the branches of study mentioned above.
2) Creation of an educational text by a way which respects chosen theory.
3) Experimental verification of suitability of the educational text from the point of view of addressees.
4) Application of chosen statistical methods.

On the basis of the analysis of the current state and setting partial aims two hypotheses were stated:

1) Educational text for students of the branch of radiographer can be made by means of application of curricular process.
2) Knowledge of students acquired by means of the elaborated educational text will have a distribution close to normal distribution.

As a convenient theory for knowledge transfer from science to educational sphere turned out to be a theory of curricular process (its applicability was accepted also by foreign scientific community, see for example impact publication (Záškodný, 2012)). The theory of curricular process was formulated by M. Pasch, T. G. Gardner, M. Certon, M. Gayl, in the world and J. Průcha, J. Brockmeyer, P. Tarábek, P. Záškodný in. Czech Republic and the Slovakia. (Veselá, 2013)

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[1] According to the European Federation of Radiographer Societies is recommended single name when referring about this profession in Europe. (European Federation of Radiographer Societies: The profession, 2012)

Methods of descriptive and mathematical statistics were used to quantify student's knowledge and verify hypotheses. A logical list of the statistical methods is: formulation of statistical inquiry, creation of scales, measurement at descriptive statistics, elementary statistical processing and assignment of theoretical distribution to empirical distribution.

On the basis of the curricular process was assembled following procedure:

1) Analysis of current scientific system (*conceptual curriculum*).
2) Transformation of the current scientific system into communicable scientific system (*intended curriculum*).
3) Creation of the educational text based on the analysis of the current scientific system (*projected curriculum*).
4) Verification of educational function of the educational text by means of experimental education of students of study branch Radiographer (*implemented curriculum-1*).
5) Creation of the educational test for quantification of acquired knowledge and its statistical processing (*implemented curriculum-2*).

C.3. THEORETICAL PART

C.3.1. CONTEMPORARY STATE OF THE NEW QUANTUM PHYSIC

C.3.1.1. The old quantum physics

The modern physics is rapidly developing from the beginning of 20th century. Into the spotlight of physicists is getting an atom and methods of its observation. The first physicist to be mentioned is Max Planck, German scientist, and his quantum hypothesis. (Krupička, 2007)

Albert Einstein applied Planck's quantum hypothesis and made a photoelectric effect clear. The experimental results of, Hungarian physicist, Phillip Lenard were explained by Einstein that the energy has to be absorbed in quanta – photons. The formula of the photoelectric phenomenon determining a kinetic energy of photoelectron T_e is:

$$T_e = \mathrm{h}\nu + W \,, \qquad [1]$$

where h is a Planck constant, ν is a frequency of photon and W is output work of an electron. (Fowler)

The photoelectric equation is very important for radiology due to the fact that it enables to calculate maximal energy of decelerating X-radiation emitted by x-ray machine. Substitution of $T_e = eU$ a $\nu = \mathrm{c}/\lambda_{\min}$ in the photoelectric equation gives us the **Duane-Hunt law:**

$$\lambda_{\min} = \frac{\mathrm{hc}}{\mathrm{e}U}, \qquad [2]$$

where λ_{min} is a minimal wave length of decelerating X-ray photon, c is the speed of light, e is elementary charge a U is a voltage between electrodes. (Navrátil & Rosina, 2005)

In 1924, Louis de Broglie stated daring idea that the matter has besides a particle property also a wave property. He stated fundamentals of wave-particle duality. The wave length of de Broglie's particle is expressed by:

$$\lambda_{DBW} = \frac{\mathrm{h}}{mv}, \qquad [3]$$

which follows from $p = \frac{E}{\mathrm{c}} = \frac{\mathrm{h}\nu}{\mathrm{c}}$, where $\nu = \frac{\mathrm{c}}{\lambda}$ a $p = mv$ (m is mass of moving particle and v its velocity). The interesting thing about de Broglie's wave is that it is moving with speed greater than light (v_{DBW} > c), because

$$v = v_{DBW}\lambda_{DBW} = \frac{mc^2}{\mathrm{h}}\frac{\mathrm{h}}{mv} = \frac{\mathrm{c}^2}{v} > 1 \qquad [4]$$

The wave-corpuscular duality enabled full description of all phenomena observed at particles and electromagnetic waves. (Beiser, Úvod do moderní fyziky, 1978) (Úlehla, Suk, & Trka, 1990)

Table 1: Summarize of various phenomenons and it possible explanation

Phenomenon	is explainable through wave properties	is explainable through particle properties
reflection	∿∿∿ ✓	●→ ✓
refraction	∿∿∿ ✓	●→ ✓
interference	∿∿∿ ✓	●→ ⊗
scattering	∿∿∿ ✓	●→ ⊗
polarization	∿∿∿ ✓	●→ ⊗
photoelectric effect	∿∿∿ ⊗	●→ ✓

source: http://hyperphysics.phy-astr.gsu.edu/hbase/mod1.html

The quantum hypothesis also inspired Holland physicist Niels Bohr for stating his model of an atom (Krupička, 2007) (the third after Thompson's and Rutherford's). (Reichel & Všetička, Bohrův model atomu, 2013) (Reichel & Všetička, Objev atomového jádra, 2013) Bohr's model of atom was based on condition that:

- Instead of a continuum of orbits, which are possible in classical mechanics, only a discrete set of circular stable orbits, called stationary states, are allowed. Atoms can exist only in certain stable states with definite energies: E_1, E_2, E_3, etc.
- The allowed (stationary) orbits correspond to those for which the orbital angular momentum of the electron is an integer multiple of $\hbar$:

$$L = n\hbar$$

This relation is known as the Bohr quantization rule of the angular momentum.

- As long as an electron remains in a stationary orbit, it does not radiate electromagnetic energy. Emission or absorption of radiation can take place only when an electron jumps from one allowed orbit to another. The radiation corresponding to the electron's transition from an orbit of energy E_n to another E_m is carried out by a photon of energy

$$\mathrm{h}\nu = E_n - E_m$$

So an atom may emit (or absorb) radiation by having the electron jump to a lower (or higher) orbit. (Zettili, 2009)

C.3.1.2. New quantum theory – Schrödinger equation

Physical quantity describing de Broglie waves is called **wave function *Ψ***. While *Ψ* itself has no physical interpretation, the square of its absolute magnitude $|\Psi|^2$ evaluated at a particular point at a particular time is proportional to the probability of experimentally finding a body there at a time. The squared wave function $|\Psi|^2$ is called **probability amplitude**. (Beiser, Úvod do moderní fyziky, 1978) (Beiser, Concepts of modern physics, 2003)

According to the geometrical definition of probability P (A) of a random event A (in this case the occurrence of particle in space) where μ (E) is a measure of space E, μ (A) is a measure of occurrence of the random event A (event A is subset E, μ (A) is probability of occurrence of particle in space E) is

$$P(\mathrm{A}) = \frac{\mu(\mathrm{A})}{\mu(\mathrm{E})}.$$

The range of probability P (A) is <0;1>. (Záškodný, Havránková, Havránek, & Vurm, 2011)

Next, suppose that the wave function *Ψ* is a complex function. Absolute value of complex number can be computed as product of a complex number *z* and a complex conjugate $\bar{z}$. The complex conjugate is given by substitution of "*i*" for "*-i*" in the general definition of a complex number. All mentioned above could be written in mathematical terms: (Hyánková & Sedláčková, 2004)

$$z = a + ib \qquad \bar{z} = a - ib \qquad |z| = \sqrt{z\bar{z}}\,.$$

State of a particle can be determined with aid of the wave function *Ψ* in the quantum mechanics. A basic type of wave function is called **Schrödinger equation** which gives us a full set of information about measurable physic quantities in current state of particle.

Suppose that we have the wave function *Ψ* in following form

$$\Psi = Ae^{-i\omega(t-x/v)}, \qquad [5]$$

substituting $\omega=2\pi\nu$ and making elementary mathematical modifications ($\lambda^{-1} = \nu/v$) we have new equation

$$\Psi = Ae^{-2\pi i(\nu t - x/\lambda)}. \qquad [6]$$

Next, suppose we have equation in form mentioned above, so that we can substitute for wave length λ and frequency v. The substitution gives us wave function Ψ dependent on energy E and momentum p,

$$v = \frac{E}{h} = \frac{E}{2\pi\hbar} \qquad \lambda = \frac{h}{p} = \frac{2\pi\hbar}{p},$$

substituting and factoring $\hbar^{-1}$ out we get

$$\Psi = Ae^{-i2\pi\left(\frac{E}{2\pi\hbar}t - \frac{p}{2\pi\hbar}x\right)}$$

$$\Psi = Ae^{-\frac{i}{\hbar}(Et-px)}. \qquad [7]$$

Equation [5] is a mathematical description of *amplitude of harmonic wave* travelling along tighten string, equation [7] is a mathematical description a *wave equivalent of free particle* with net energy E and momentum p. (Beiser, Úvod do moderní fyziky, 1978) (Úlehla, Suk, & Trka, 1990)

Particles are not always free in real world that means it would be advantageous to find differential form of the wave equation applicable for different circumstances. (Skála, 2005)

Derivation of non-stationary and stationary Schrödinger equation: (Beiser, Úvod do moderní fyziky, 1978) (Úlehla, Suk, & Trka, 1990) (Beiser, Concepts of modern physics, 2003)

Firstly, we derive equation [7] twice by x:

$$\frac{\partial\Psi}{\partial x} = Ae^{-\frac{i}{\hbar}(Et-px)}.i\frac{p}{\hbar}$$

$$\frac{\partial^2\Psi}{\partial x^2} = Ae^{-\frac{i}{\hbar}(Et-px)}.\left(i^2\frac{p^2}{\hbar^2}\right) = \Psi\left(-\frac{p^2}{\hbar^2}\right). \qquad [8]$$

From equation [8] we express $p^2\Psi$

$$p^2\Psi = -\hbar^2\frac{\partial^2\Psi}{\partial x^2}. \qquad [9]$$

Next, we derive equation [7] by time t:

$$\frac{\partial\Psi}{\partial t} = Ae^{-\frac{i}{\hbar}(Et-px)}.\left(-\frac{iE}{\hbar}\right) = \Psi\left(-\frac{iE}{\hbar}\right). \qquad [10]$$

Again, using equivalent mathematical operations we express $E\Psi$

$$E\Psi = -\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = i\hbar\frac{\partial\Psi}{\partial t}. \qquad [11]$$

Net energy E could be written in form of equation [12] at low velocities compared with the speed of light ($v \ll c$)

$$E = \frac{1}{2}mv^2 + V = \frac{p^2}{2m} + V. \qquad [12]$$

We extend the formula for kinetic energy [12] by “one”, in other words by m/m, and substitute $(mv)^2$ for quadrate of momentum p^2.

Multiplying equation [12] by wave equation Ψ we get

$$E\Psi = \frac{p^2\Psi}{2m} + V\Psi \, . \qquad [13]$$

Substituting equations [9] and [11] into equation [13] we get the **non-stationary Schrödinger equation:**

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi \, . \qquad [14]$$

The three dimensional form of the non-stationary Schrödinger equations is given by substitution of $\frac{\partial^2\Psi}{\partial x^2}$ for quadrate of the Laplace operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

The energy of a particle is not strictly time-dependent, potential energy of a particle V depends on position. Due to the fact, the Schrödinger equation can be transformed into equation dependent on the position.

The one dimensional wave function Ψ of the form

$$\Psi = Ae^{-\frac{i}{\hbar}(Et-px)} \, .$$

Using the known mathematical formula $a^{r+s} = a^r . a^s$ we get (Mikulčák, 2006):

$$\Psi = Ae^{-\frac{iEt}{\hbar}} e^{\frac{ipx}{\hbar}} \, ,$$

substituting $e^{\frac{ipx}{\hbar}}$ for ψ we get wave function in the form of

$$\Psi = e^{-\frac{iEt}{\hbar}}\psi \, , \qquad [15]$$

ψ is called by **function of location** in this case.

Substituting equation [15] for [13] we get the **one dimensional stationary Schrödinger equation.**

$$-Ee^{-\frac{iEt}{\hbar}}\psi = \frac{\hbar^2}{2m}\frac{\partial^2 e^{-\frac{iEt}{\hbar}}\psi}{\partial x^2} - Ve^{-\frac{iEt}{\hbar}}\psi \, .$$

At last, we have to transform the equation to more pleasant form. Reducing the equation by $e^{-\frac{iEt}{\hbar}}$ and transferring of all constituents on the left hand side, we get:

$$-E\psi = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - V\psi$$

$$0 = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi - V\psi$$

$$0 = \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (E - V)\psi$$

$$-\frac{2m}{\hbar^2}(E-V)\psi = \frac{\partial^2\psi}{\partial x^2}.$$

The stationary Schrödinger equation acquires form of

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0. \qquad [16]$$

The three dimensional stationary Schrödinger equation is given by substituting the second partial derivation of function of position with respect to x $\frac{\partial^2\psi}{\partial x^2}$ for quadrate of the Laplace operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

C.3.1.3. The New Quantum theory – operational calculus

Dynamic quantities as momentum p or net energy E are expressed by **operators** in the quantum mechanics. The operator can be easily characterized as instruction needs to be done, so as we could go from one physical quantity to another. (Záškodný, Kurikulární proces fyziky (s přehledem základů teoretické fyziky), 2009)

Due to the Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \hbar \quad \wedge \quad \Delta E \Delta t \geq \hbar$$

we cannot compute expected values of $\bar{p}$ and $\bar{E}$ by means of classical formulas for the expected values of an arbitrary quantity

$$\bar{G}(x) = \int_{-\infty}^{\infty} \Psi^* G(x) \Psi \, dx$$

which would be for $\bar{p}$ and $\bar{E}$ (Beiser, Úvod do moderní fyziky, 1978)

$$\bar{p} = \int_{-\infty}^{\infty} \Psi^* p \Psi \, dx$$

$$\bar{E} = \int_{-\infty}^{\infty} \Psi^* E \Psi \, dx.$$

Definition of operator calculus, derivation of non-stationary Schrödinger equation: (Beiser, Úvod do moderní fyziky, 1978)

When we derivate operator for momentum p and energy E we come out from equation of the wave function [7]. Final formulas for operators we get by partial derivation of the wave function with respect to distance x (for p) and with respect to time t (for E).

$$\frac{\partial\Psi}{\partial x} = Ae^{-\frac{i}{\hbar}(Et-px)}.i\frac{p}{\hbar} \quad \Rightarrow \quad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad [17]$$

$$\frac{d\Psi}{dt} = Ae^{-\frac{i}{\hbar}(Et-px)}.\left(-\frac{iE}{\hbar}\right) \quad \Rightarrow \quad \hat{E} = i\hbar\frac{\partial}{\partial t} \qquad [18]$$

The operators enable to use trivial derivation of the non-stationary Schrödinger equation. The process is following:

1) We proceed from the equation of net mechanic energy E

$$E = T + V;$$

2) To the formula for kinetic energy $T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ we substitute a operator for quadrate of momentum p^2 [17]

$$T = \frac{\hat{p}^2}{2m} \quad \Rightarrow \quad T = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2},$$

that gives us the final formula:

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}; \qquad [19]$$

3) We substitute the formulas for operators $\hat{E}$ and $\hat{T}$ for E and T into the equation in the step number 1

$$i\hbar\frac{\partial}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V;$$

4) Multiplying the equation by Ψ in the third step, we get non-stationary Schrödinger equation.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi.$$

C.3.1.4. Importance of quantum mechanics for magnetic resonance

Without overstating, we can say that magnetic resonance would not exist without development of the quantum theory and quantum mechanics. The development of quantum mechanics enabled acquisition of tools for describing of states of the quantum objects. This opened the gateway to a deeper understanding of the physical nature of the world.

The three dimensional description of the electron in orbital of hydrogen atom is identical to the equation [16]

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0,$$

where V is electrostatic potential energy ($V=e^2/4\pi\varepsilon_0 r$). Due to the symmetry of the hydrogen atom is advantageous to transform Schrödinger equation into the spherical coordinates.

Spherical coordinate system is determined by three coordinates:

- **co-altitude *r***, $r = \sqrt{x^2 + y^2 + z^2}$,
- **zenith angle *ϑ*** is between position vector r and the positive part of z-axis, its value is computed according to $\vartheta = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$,
- **inclination angle *φ*** is between the projection of position vector r into xy- plane and the positive part of x-axis, $\operatorname{arctg}\left(\frac{x}{y}\right)$.

C.3.2. TRANSFORMATION OF THE LAPLACE OPERATOR

Sources (The Laplacian Operator from Cartesian to Cylindrical to Spherical Coordinates)**,** (Turrell)**,** and (The Laplacian in Terms of Polar Coordinates) were used for description of the derivation.

The transformation of the Schrödinger equation will be done using the chain rule which takes for *x* coordinate the form of:

$$\frac{\partial}{\partial x}=\left(\frac{\partial r}{\partial x}\right)\frac{\partial}{\partial r}+\left(\frac{\partial \vartheta}{\partial x}\right)\frac{\partial}{\partial \vartheta}+\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial}{\partial \varphi},$$

similarly for *y* and *z*.

1) Derivation of position vector *r* with respect to *x*, *y*, *z*

$$\frac{\partial r}{\partial x}=\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}2x=\frac{x}{r}=\sin\vartheta\cos\varphi$$

$$\frac{\partial r}{\partial y}=\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}2y=\frac{y}{r}=\sin\vartheta\sin\varphi$$

$$\frac{\partial r}{\partial z}=\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}2z=\frac{z}{r}=\cos\vartheta$$

Table 2: Summarize of the important conversion formulas of Cartesian coordinate system to spherical coordinate system.

spherical → Cartesian	Cartesian → spherical	
x= r sin ϑ cos φ	$r=\sqrt{x^2+y^2+z^2}$	
y=r sin ϑ sin φ	$\cos\vartheta=\frac{z}{\sqrt{x^2+y^2+z^2}}$	$\cos\varphi=\frac{x}{\sqrt{x^2+y^2}}$
z=r cos ϑ	$\sin\vartheta=\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$	$\sin\varphi=\frac{y}{\sqrt{x^2+y^2}}$

source: http://skisickness.com/2009/11/20/

2) Derivation of zenith angle ϑ with respect to x

$$\frac{\partial}{\partial x}\arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=-\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\cdot\frac{\partial}{\partial x}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=$$

$$=\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}.z\left(\frac{1}{2}\frac{1}{\left(\sqrt{x^2+y^2+z^2}\right)^3}\right)\frac{\partial}{\partial x}\left(x^2+y^2+z^2\right)=$$

$$=\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\cdot\frac{2xz}{2\left(\sqrt{x^2+y^2+z^2}\right)^3}=\frac{xz}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

Modification of the 1st. expression under square root

$$\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}=\sqrt{1-\frac{z^2}{x^2+y^2+z^2}}=\sqrt{\frac{x^2+y^2+z^2-z^2}{x^2+y^2+z^2}}=\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}$$

Mathematical modification of the result of derivation of the zenith angle

$$\frac{xz}{\left(\frac{x^2+y^2}{x^2+y^2+z^2}\right)^{\frac{1}{2}}\left(\sqrt{x^2+y^2+z^2}\right)^{\frac{3}{2}}}=\frac{xz}{\sqrt{x^2+y^2}\left(\sqrt{x^2+y^2+z^2}\right)^2}=\frac{\cos\vartheta\cos\varphi}{r}$$

3) Derivation of the zenith angle with respect to y (the same steps like latter)

$$\frac{\partial}{\partial y}\arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=\frac{yz}{\sqrt{x^2+y^2}\left(\sqrt{x^2+y^2+z^2}\right)^2}=\frac{\cos\vartheta\sin\varphi}{r}$$

4) Derivation of the zenith angle ϑ with respect to z

$$\frac{\partial}{\partial z}\arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=-\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\cdot\frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=$$

$$=-\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\cdot\left(\frac{\sqrt{x^2+y^2+z^2}-\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}z\frac{\partial}{\partial z}\left(x^2+y^2+z^2\right)}{\left(\sqrt{x^2+y^2+z^2}\right)^2}\right)=$$

$$=\frac{1}{\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\cdot\frac{\frac{2z^2}{2\sqrt{x^2+y^2+z^2}}-\sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2}=$$

$$=\frac{\frac{z^2}{\sqrt{x^2+y^2+z^2}}-\sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2\sqrt{1-\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)^2}}\Rightarrow\frac{\frac{z^2}{\sqrt{x^2+y^2+z^2}}-\sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}}}\Rightarrow$$

$$\Rightarrow\frac{-\sqrt{x^2+y^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

5) Derivation of the inclination angle φ with respect to x, y, z

$$\frac{\partial}{\partial x}\arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right)\Rightarrow\frac{\partial\varphi}{\partial x}=\frac{\sin\varphi}{\sqrt{x^2+y^2}}=\frac{\sin\varphi}{r\sin\vartheta}$$

$$\frac{\partial}{\partial y}\arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right)\Rightarrow\frac{\partial\varphi}{\partial y}=\frac{\cos\varphi}{\sqrt{x^2+y^2}}=\frac{\sin\varphi}{r\sin\vartheta}$$

$$\frac{\partial}{\partial z}\arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right)\Rightarrow\frac{\partial\varphi}{\partial z}=0$$

6) Application of the chain rule

$$\frac{\partial}{\partial x}=\left(\frac{\partial r}{\partial x}\right)\frac{\partial}{\partial r}+\left(\frac{\partial \vartheta}{\partial x}\right)\frac{\partial}{\partial \vartheta}+\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial}{\partial \varphi}=\sin\vartheta\cos\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial}{\partial \vartheta}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}$$

$$\frac{\partial}{\partial y}=\left(\frac{\partial r}{\partial y}\right)\frac{\partial}{\partial r}+\left(\frac{\partial \vartheta}{\partial y}\right)\frac{\partial}{\partial \vartheta}+\left(\frac{\partial \varphi}{\partial y}\right)\frac{\partial}{\partial \varphi}=\sin\vartheta\sin\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial}{\partial \vartheta}+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}$$

$$\frac{\partial}{\partial z}=\left(\frac{\partial r}{\partial z}\right)\frac{\partial}{\partial r}+\left(\frac{\partial \vartheta}{\partial z}\right)\frac{\partial}{\partial \vartheta}+\left(\frac{\partial \varphi}{\partial z}\right)\frac{\partial}{\partial \varphi}=\cos\vartheta\frac{\partial}{\partial r}-\frac{\sin\vartheta}{r}\frac{\partial}{\partial \vartheta}$$

7) Calculation of the 2nd derivations $\frac{\partial^2}{\partial x^2},\frac{\partial^2}{\partial y^2},\frac{\partial^2}{\partial z^2}$

$$\frac{\partial^2}{\partial x^2}=\left(\sin\vartheta\cos\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial}{\partial \vartheta}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)$$

$$\left(\sin\vartheta\cos\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial}{\partial \vartheta}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)=\sin\vartheta\cos\varphi$$

$$\left(\sin\vartheta\cos\varphi\frac{\partial^2}{\partial r^2}+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial^2}{\partial r\partial\vartheta}-\frac{\cos\vartheta\cos\varphi}{r^2}\frac{\partial}{\partial \vartheta}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial r\partial\varphi}+\frac{\sin\varphi}{r^2\sin\vartheta}\frac{\partial}{\partial \varphi}\right)+$$

$$\frac{\cos\vartheta\cos\varphi}{r}\left(\sin\vartheta\cos\varphi\frac{\partial^2}{\partial r\partial\vartheta}+\cos\vartheta\cos\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial^2}{\partial \vartheta^2}-\frac{\sin\vartheta\cos\varphi}{r}\frac{\partial}{\partial \vartheta}-\right.$$

$$\left.-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial\varphi\partial\vartheta}+\frac{\cos\vartheta\sin\varphi}{r\sin^2\vartheta}\frac{\partial}{\partial \varphi}\right)-\frac{\sin\varphi}{r\sin\vartheta}\left(\sin\vartheta\cos\varphi\frac{\partial^2}{\partial r\partial\varphi}-\sin\vartheta\sin\varphi\frac{\partial}{\partial r}+\right.$$

$$\left.+\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial^2}{\partial\varphi\partial\vartheta}-\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial}{\partial \vartheta}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial \varphi^2}-\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)$$

$$\frac{\partial^2}{\partial y^2}=\left(\sin\vartheta\sin\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial}{\partial \vartheta}+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)$$

$$\left(\sin\vartheta\sin\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial}{\partial \vartheta}+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)=$$

$$\sin\vartheta\sin\varphi\left(\sin\vartheta\sin\varphi\frac{\partial^2}{\partial r^2}+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial^2}{\partial r\partial\vartheta}+\frac{\cos\vartheta\sin\varphi}{r^2}\frac{\partial}{\partial \vartheta}+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial r\partial\varphi}+\frac{\cos\varphi}{r^2\sin\vartheta}\frac{\partial}{\partial \varphi}\right)+$$

$$+\frac{\cos\vartheta\sin\varphi}{r}\left(\sin\vartheta\sin\varphi\frac{\partial^2}{\partial\vartheta\partial r}+\cos\vartheta\sin\varphi\frac{\partial}{\partial r}+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial^2}{\partial \vartheta^2}-\frac{\sin\vartheta\sin\varphi}{r}\frac{\partial}{\partial \vartheta}+\right.$$

$$\left.+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial\vartheta\partial\varphi}+\frac{\cos\varphi\cos\vartheta}{r\sin^2\vartheta}\frac{\partial}{\partial \varphi}\right)+\frac{\cos\varphi}{r\sin\vartheta}\left(\sin\vartheta\sin\varphi\frac{\partial^2}{\partial\varphi\partial r}+\sin\vartheta\cos\varphi\frac{\partial}{\partial r}+\right.$$

$$\left.+\frac{\cos\vartheta\sin\varphi}{r}\frac{\partial^2}{\partial\varphi\partial\vartheta}++\frac{\cos\vartheta\cos\varphi}{r}\frac{\partial}{\partial \vartheta}+\frac{\cos\varphi}{r\sin\vartheta}\frac{\partial^2}{\partial \varphi^2}-\frac{\sin\varphi}{r\sin\vartheta}\frac{\partial}{\partial \varphi}\right)$$

$$\frac{\partial^2}{\partial z^2}=\left(\cos\vartheta\frac{\partial}{\partial r}-\frac{\sin\vartheta}{r}\frac{\partial}{\partial\vartheta}\right)\left(\cos\vartheta\frac{\partial}{\partial r}-\frac{\sin\vartheta}{r}\frac{\partial}{\partial\vartheta}\right)=\cos^2\vartheta\frac{\partial^2}{\partial r^2}-$$
$$-\cos\vartheta\sin\vartheta\left(\frac{1}{r}\frac{\partial^2}{\partial r\partial\vartheta}-\frac{1}{r^2}\frac{\partial}{\partial\vartheta}\right)-\frac{\sin\vartheta}{r}\left(\text{os}\,\vartheta\frac{\partial^2}{\partial\vartheta\partial r}-\sin\vartheta\frac{\partial}{\partial r}\right)+$$
$$+\frac{\sin\vartheta}{r^2}\left(\sin\vartheta\frac{\partial^2}{\partial\vartheta^2}+\cos\vartheta\frac{\partial}{\partial\vartheta}\right)$$

The squared Laplace operator ∇^2 will be given according to $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

The sum of the 2nd partial derivations enables us to simplify sophisticated mathematic expression by means of basic mathematical operations (ex. factoring out) and formulas (ex. $\sin^2\alpha+\cos^2\alpha=1$).

We will multiply and reorder given mathematical expression according to the partial derivation they include.

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\left(\sin^2\vartheta\left(\sin^2\varphi+\cos^2\varphi\right)+\cos^2\vartheta\right)\frac{\partial^2}{\partial r^2}+$$
$$+\left(\frac{\cos^2\vartheta\cos^2\varphi+\sin^2\varphi\cos^2\vartheta+\sin^2\vartheta}{r^2}\right)\frac{\partial^2}{\partial\vartheta^2}+$$
$$+\left(\frac{\sin^2\varphi+\cos^2\varphi}{r^2\sin^2\vartheta}\right)\frac{\partial^2}{\partial\varphi^2}+\left(\frac{\left(\sin^2\varphi+\cos^2\varphi\right)+\left(\sin^2\varphi+\cos^2\varphi\right)\cos^2\vartheta+\sin^2\vartheta}{r}\right)\frac{\partial}{\partial r}+$$
$$+\left(\frac{\left(\sin^2\varphi+\cos^2\varphi\right)\cos\vartheta}{r^2\sin\vartheta}+\frac{2\sin\vartheta\sin^2\varphi\cos\vartheta-2\sin\vartheta\sin^2\varphi\cos\vartheta}{r^2}\right)\frac{\partial}{\partial\vartheta}+$$
$$+\left(\frac{2\sin\varphi\cos\varphi\left(\sin^2\vartheta+\cos^2\vartheta\right)-2\sin\varphi\cos\varphi}{r^2\sin^2\vartheta}\right)\frac{\partial}{\partial\varphi}+$$
$$+\left(\frac{\left(\sin^2\varphi+\cos^2\varphi\right)2\sin\vartheta\cos\vartheta-2\cos\vartheta\sin\vartheta}{r}\right)\frac{\partial^2}{\partial r\partial\vartheta}+$$
$$+\left(\frac{2\sin\vartheta\sin\varphi\cos\varphi-2\sin\vartheta\sin\varphi\cos\varphi}{r\sin\vartheta}\right)\frac{\partial^2}{\partial r\partial\varphi}+$$
$$+\left(\frac{2\sin\varphi\cos\vartheta\cos\varphi-2\sin\varphi\cos\vartheta\cos\varphi}{r^2\sin\vartheta}\right)\frac{\partial^2}{\partial\varphi\partial\vartheta}$$

After making the expression simpler and its regrouping we will get:

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\vartheta^2}+\frac{\cos\vartheta}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}+\frac{1}{r^2\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}.$$

The Laplace operator in spherical coordinates is often written in the form of

$$\nabla^2 = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right] + \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{r^2\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}.$$

The Schrödinger equation for electron of the hydrogen atom is

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)\right] + \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\psi}{\partial\vartheta}\right) + \frac{1}{r^2\sin^2\vartheta}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0,$$

by substituting for potential energy V and multiplication of whole equation $r^2 sin^2\vartheta$ we will get

$$\sin^2\vartheta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \sin\vartheta\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\psi}{\partial\vartheta}\right) + \frac{\partial^2\psi}{\partial\varphi^2} + \frac{2mr^2\sin^2\vartheta}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right)\psi = 0.. \qquad [20]$$

The Schrödinger equation in spherical coordinates makes transformation of the equation [20] to the product of three different expressions dependent each only on one variable possible, in other words, Schrödinger equation takes the form of: (Beiser, Úvod do moderní fyziky, 1978)

$$\psi(r,\vartheta,\varphi) = R(r)\Theta(\vartheta)\Phi(\varphi).$$

Separation of variables: (Beiser, Úvod do moderní fyziky, 1978)

The separation begins with substitution of $\psi = R\Phi\Theta$ to equation [20] and therefore dividing by $R\Phi\Theta$. After modifications we will get

$$\frac{\sin^2\vartheta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{\sin\vartheta}{\Theta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right) + \frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{2mr^2\sin^2\vartheta}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right) = 0.$$

We will subtract $\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}$ from the latter equation and we will get

$$\frac{\sin^2\vartheta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{\sin\vartheta}{\Theta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right) + \frac{2mr^2\sin^2\vartheta}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right) = -\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}. \qquad [21]$$

To make equation [21] valid, both sides of the equation have to be equal to the same constant. The constant is marked m_l and equals to

$$m_l^2 = -\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}. \qquad [22]$$

By division of the equation [21] by $sin^2\vartheta$ we will get

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{1}{\Theta\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right) + \frac{2mr^2}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right) = \frac{m_l^2}{\sin^2\vartheta},$$

subtracting the 2nd constituent of the polynomial on the left hand side will result in

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2mr^2}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right) = \frac{m_l^2}{\sin^2\vartheta} - \frac{1}{\Theta\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right), \qquad [23]$$

where each side of the equation depends on different variable. The equation will be valid, if both sides equal to $l(l+1)$.

Differential equations [22] a [23] are usually written in form of

$$\frac{d^2\Phi}{d\varphi^2}+m_l^2\Phi=0\,, \qquad [24]$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\left[\frac{2m}{\hbar^2}\left(E+\frac{e^2}{4\pi\varepsilon_0 r}\right)-\frac{l(l+1)}{r^2}\right]R=0\,, \qquad [25]$$

$$\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Theta}{\partial\vartheta}\right)+\left[l(l+1)-\frac{m_l^2}{\sin^2\vartheta}\right]\Theta=0\,. \qquad [26]$$

We will state **the full set of quantum number of the electron in hydrogen atom** with the aid of the latter mentioned equation. The full set of quantum numbers describes the state of electron in atomic orbital. Similar set of quantum numbers can be derived for nucleons in nucleus of atom. (Janeček, Klaus, & Hřivnák, Modely jader, 2006) (Janeček, Klaus, & Hřivňák, Kvantová čísla, 2006)

Fermions[2] are subject to the **Pauli's exclusion principle** which says that *in the state defined by particular values of quantum numbers of the total set of quantum numbers, there may be at most one fermion*. (Záškodný & Procházka, Survey of Principles of Theoretical Physics (with application to radiology), 2014)

Solving the differential equation [24] results in

$$\Phi(\varphi)=Ae^{im_l\varphi}\,, \qquad [27]$$

where A is a constant of integration. The equation [27] has to give only one value for the determined point in space so must be truth that:

$$\Phi(\varphi)=Ae^{im_l\varphi}\Rightarrow\Phi(\varphi)=Ae^{im_l(\varphi+2\pi)}.$$

The condition will be truth, if $m_l\in\mathbb{Z}$. The constant m_l is **called the magnetic quantum number** which determines the orientation of momentum vector of electron. The magnetic quantum number gets values from interval of (-*l*…0…+*l*). (Beiser, Úvod do moderní fyziky, 1978)

The electron orbiting the nucleus makes little current loop with magnetic moment μ. The moment of force having effect on electron is given by

$$\tau=\mu\times B=\mu B\sin\vartheta\,,$$

where ϑ is angle between magnetic moment μ and magnetic induction B. If we choose the reference frame where the magnetic potential energy V_m equal to zero for $\vartheta=\pi/2$, then for each change of orientation of the magnetic moment μ in an external magnetic field B must perform the work W, which is equal to the change in potential energy according to the formula

---

[2] Between fermions belongs leptons (ex. electrons), hadrons (ex. protons, neutrons - nukleons), baryons (all hadrons which are fermions as well). (Janeček, Klaus, & Hřivňák, Třídění elementárních částic, 2006)

$$V_m = -W = -\int_{\pi/2}^{\vartheta} \tau \, d\vartheta = -\mu B \int_{\pi/2}^{\vartheta} \sin\vartheta \, d\vartheta = -\mu B \cos\vartheta \, . \qquad [28]$$

Magnetic moment of the electron is determined by

$$\mu = iA = \frac{q}{t}\pi r^2 \, ,$$

if we substitute $t = \frac{2\pi r}{v}$ and if we extend fraction by "one", in other word by $\frac{m}{m}$, we will get

$$\mu = \frac{q}{2m} mvr = \frac{q}{2m} L \, , \qquad [29]$$

where *L* is angular momentum of electron. The fraction $\frac{q}{2m}$ is called **gyromagnetic ratio** and is marked as *g*. (Beiser, Úvod do moderní fyziky, 1978) (Brown, 2014)

By substitution of equation [28] into equation [29] we will get formula for potential energy

$$V_m = -\mu B \cos\vartheta = -\frac{q}{2m} LB \cos\vartheta \, . \qquad [30]$$

As well as the orbital momentum *L* is quantized, so that the direction of the angular momentum vector *L* is quantized with respect to external field (spatial quantization). Only the *z* component of *L* is quantized according to

$$L_z = m_l \hbar \, .$$

The number of possible orientations of *z* component of angular momentum vector L_z comes out from valid values magnetic quantum number m_l. Components L_x and L_y are not quantized. The vector of angular momentum makes precession (traces out a cone). The motion is also called as Larmor precession.

Solution of the differential equation [25] for radial constituent *R(r)* is very sophisticated and it is expressed by the Laguerre polynomials which can be solved for $E \geq 0$ (ionized hydrogen atom). For $E_n < 0$ (bounded electron) has the solution the same form as the formula for energetic levels of Bohr's atomic model:

$$E_n = -\frac{me^4}{32\pi^2 \varepsilon_0^2 \hbar^2}\left(\frac{1}{n^2}\right),$$

where $n \in \mathbb{Z}$. Next necessary condition to solve the equation [25] is that the **principal quantum number** *n* determining quantization of energy of the electron has to satisfy the condition that $n \geq (l+1)$. (Beiser, Úvod do moderní fyziky, 1978)

Solution of the last differential equation [26] for $\Theta(\vartheta)$ is expressed by the Legendre polynomials and is very complicated. It is important to notice that the polynomials, mentioned latter, exists only for $l \in \mathbb{Z} \wedge l \geq |m_l|$. The constant *l* is called **orbital quantum number**. The azimuthal quantum number describes quantization of angular momentum according to the expression

$$L = \sqrt{\left[l(l+1)\right]}\hbar .$$

The last quantum number describing quantization of intrinsic angular momentum is **spin magnetic quantum number** m_s. Mathematical derivation of the spin quantum number (for example from Klein-Gordon equation) is beyond the scope of this work. Let us consider, therefore, for evidence of the following facts ex. Stern-Gerlach experiment or the Einstein-de Hass experiment. (Janeček, Klaus, & Hřivňák, Kvantová čísla, 2006)

The *z* component of intrinsic angular momentum of electron is quantized in the same manner as the *z* component of orbital angular momentum L_z by formula

$$S_z = m_s\hbar = \pm\frac{1}{2}\hbar .$$

The magnetic spin quantum number m_s takes only values $m_s = \pm½$, which follows from the Dirac theory and the same conclusions can be reached by examining the spectral lines.

The quantity of intrinsic angular momentum of electron *S* is given by:

$$S = \sqrt{\left[s(s+1)\right]}\hbar .$$

The gyromagnetic ratio for electron spin is almost double and due to the fact is equation [30]:

$$\mu_s = -\frac{q}{m}S \quad \Rightarrow \quad V_m = -\frac{q}{m}SB\cos\vartheta$$

which is expression for potential energy of electron V_m. (Beiser, Úvod do moderní fyziky, 1978) (Úlehla, Suk, & Trka, 1990)

Latter in the text was mentioned that the set of the quantum numbers characterize the state of the electron in atomic orbital. It remains to answer the question: **What is the atomic orbital?** The atomic orbital is nothing other than the electron wave function ψ^2. (Mička & Lukeš, 2009)

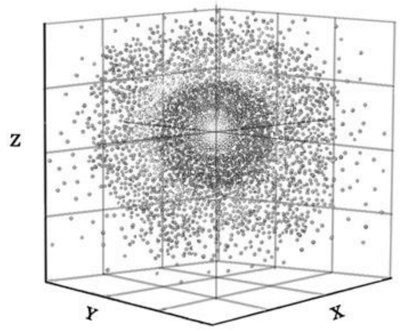


Figure 1: The electron cloud of hydrogen atom.
source: http://www.pozitivne.cz/sites/default/files/field/image/pr0016_1_allatra_elektronovy-oblak-atomu-vodiku.jpg

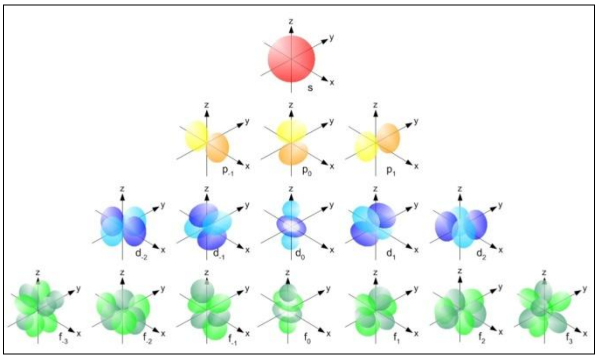


Figure 2: Overview of the atomic orbitals.
source: http://chemwiki.ucdavis.edu/@api/deki/files/8855/Single_electron_orbitals.jpg

C.3.3. APPLICATION OF QUANTUM MECHANICS IN MR

The solution of the stationary Schrödinger equation and finding the particular values of quantum numbers of the total set of quantum numbers *n*, *l*, m_l, m_s made explanation of the Larmor precession possible for electron in atomic orbital as well as for protons and neutrons in atomic nucleus.

The nucleons in atomic nucleus occupy, similarly as electrons, nuclear orbitals. Its value and order is different from those for electron, though. The model is called **nuclear shell model**.

The nuclear shell model is not strictly empiric model. Approximate energetic spectrum can be obtained from quantum mechanical calculations.

The calculations are based on the Schrödinger equation in which is nuclear interaction approximately determined by effective potential. (Janeček, Klaus, & Hřivnák, Modely jader, 2006)By substitution of μ in equation [29] to equation for torque τ we will get (vector cross product is expressed by sin ϑ)

$$\tau = \vec{\mu} \times \vec{B} = \frac{q}{2m} LB \sin \vartheta ,$$

consequently, using formula $s = r\vartheta$, where *s* can be substituted for $d\mu$ and *r* for μ sin ϑ, we will derive formula for Larmor angular frequency Ω

$$d\vec{\mu} = \mu \sin \vartheta \, d\vartheta = \gamma \mu B_0 \sin \vartheta \, dt \quad \Rightarrow \quad \Omega = \frac{d\vartheta}{dt} = \gamma B_0 . \qquad [31]$$

From equation [31] it is obvious that the angular Larmor frequency is depended only on gyromagnetic ratio of proton and magnetic induction of outer magnetic field. (Brown, 2014).

Hydrogen atoms are mainly used at magnetic resonance imaging due to their major volume in human body. Human body is formed from ⅔ of water (H_2O).

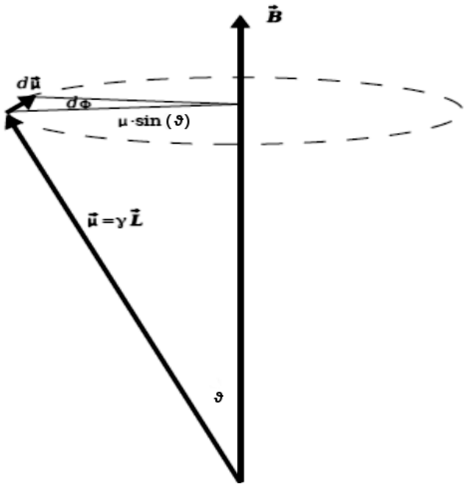


Figure 3: Illustration of the precession (customized)
source: Brown, Keith. The Magnetic Moment and the Bloch Equations. [Online] [Citace: 3. březen 2014.] http://chem4823.usask.ca/nmr/Bloch.pdf

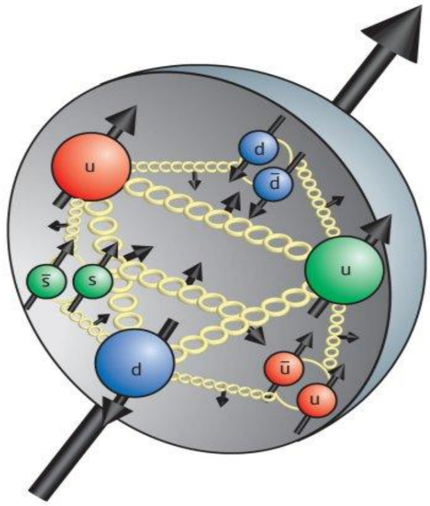


Figure 4: Illustration of proton spin and its constituents.
source: http://sciencespot.co.uk/images/SPT78-proton.jpg

This property of human body makes MR highly sensitive (primarily) for soft tissue imaging. The hydrogen has convenient attributes for MR imaging. The vector of magnetic moment $\vec{\mu}$ is detectable only at nucleuses with unpaired nucleons (^{1}H, ^{2}H, ^{31}P, ^{13}C…), because of its mutual compensation of magnetic moment vector $\vec{\mu}$ (^{4}He, ^{12}C…). (Hornak, Spin physics, 2010)

For calculating the Larmor frequency υ_0, we have to divide equation [31] by 2π (the nature of this step comes from $\nu_0 = \Omega/2\pi$). Therefore, expression for Larmor frequency is

$$\nu_0 = \frac{\gamma B_0}{2\pi}. \qquad (32)$$

The energy of emitted radiofrequency photon which excites hydrogen nucleuses, is derived from the Planck's formula for energy of photon

$$E = \mathrm{h}\nu_0 = \frac{\mathrm{h}\gamma B_0}{2\pi} = \hbar\gamma B_0. \qquad (33)$$

Description of principle of magnetic resonance imaging is in scope of the educational text (projected curriculum) which was translated into English and proceeded in OEDM-SERM 13/14, the division of education and curricular process (available at: http://www.oedm-serm.org/wp-content/uploads/2014/08/001.pdf).

C.4. HYPOTHESES AND RESEARCH METHODOLOGY

C.4.1. OPERATING HYPOTHESES AND OBJECTIVES

The hypotheses of author's work are:

1) Educational text for students of the branch of radiographer can be made by means of application of curricular process.

2) Knowledge of students acquired by means of the elaborated educational text will have a distribution close to normal distribution.

Besides the hypothesis, the author's was also developer with following objectives:

1) Selection of a theory of knowledge transfer to the level of the branches of study mentioned above.

2) Creation of an educational text by a way which respects chosen theory.

3) Experimental verification of suitability of the educational text from the point of view of addressees.

4) Application of chosen statistical methods.

C.4.2. THE THEORY OF CURRICULAR PROCESS

One of the objectives is selection of the theory of knowledge transfer. As a convenient theory for knowledge transfer from science to educational sphere turned out to be a theory of curricular process (its applicability was accepted also by foreign scientific community, see for example impact publication (Záškodný, 2012)).

The curricular process can be defined as array of transforming variant forms of curriculum. The curriculum is, simply, the content of education. (Průcha, 2013) Individual variant forms of curriculum are: conceptual curriculum, indented curriculum, projected curriculum, implemented curriculum-1, implemented curriculum-2 and applied curriculum.

Individual didactic transformation of variant forms of curriculum is in the scope of a didactic communication. The didactic communication in physics is consistent process of transfer of results and methods of scientific knowledge to the mind of individuals who did not participate in creation of the knowledge.

Content of education has to undergo communication, content, curricular, educational and application transformation. Each didactic transformation is marked as T^1-T^5 (Záškodný, 2009). Unification of the curricular process of physics with didactic transformations could be summarized by figure 5.

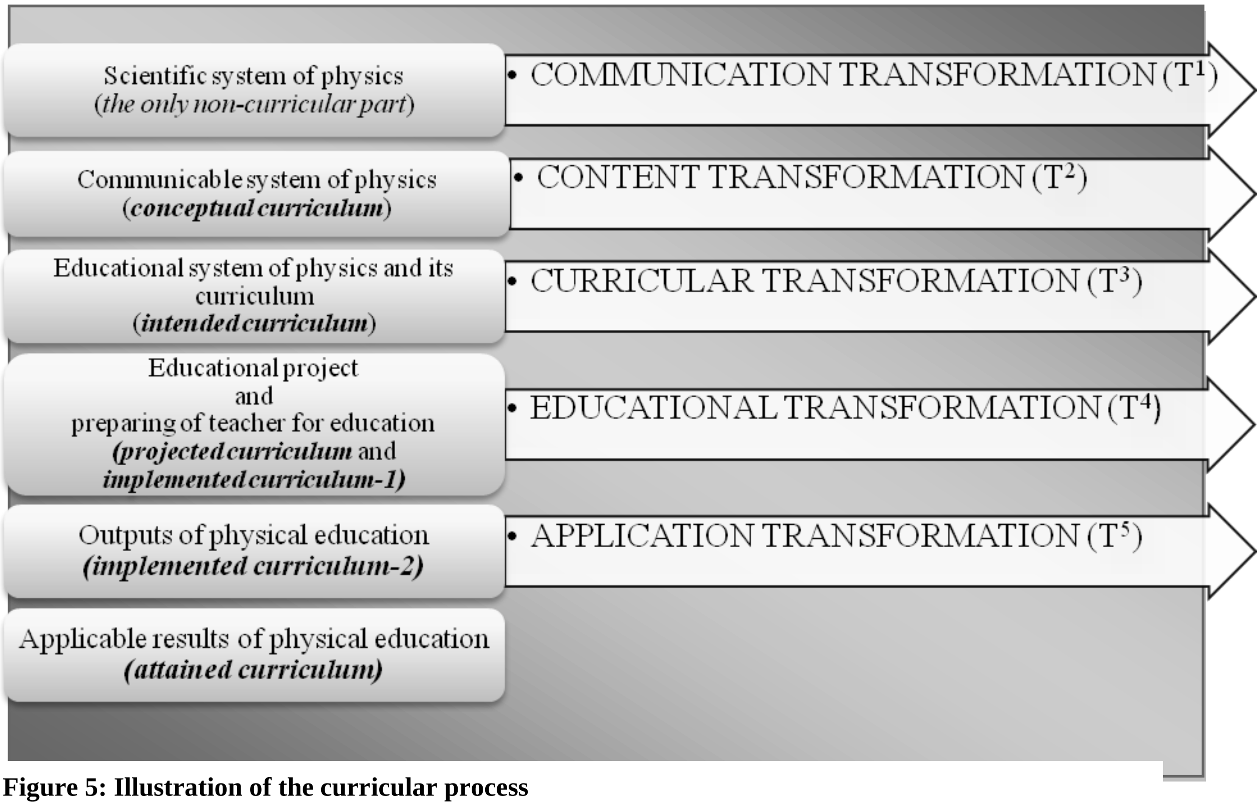


Figure 5: Illustration of the curricular process
source: (Záškodný, Kurikulární proces fyziky (s přehledem základů teoretické fyziky), 2009).

Ad 1. – ***conceptual curriculum*** (see Figure 6)

The quantum non-statistical physics is field of physics interested in phenomenons at microscopic level which are not observable by means of senses. It is necessary to use devices. Classical going (phenomenon → image → concept → mathematical relation → experiment → application) is not usable. Due to the fact, we have to use different process:

a) **Phenomenon** connected with microobject (not directly observable),
b) **Experiment** (information about microobject are acquired using devices),
c) **Mathematical model** (result of experiment given into mathematical context),
d) **Image** (created without direct contact with phenomenon),
e) **Conception** (based on experiences from macroworld),
f) **Application.**

Quantum object and effect (e.g. electron and its states) cannot be examined without using instruments.
Use of instrument – e.g. finding of spectral series a) of hydrogen atom, b) of other atoms

Difference between quantum and classical mechanics for small quantum numbers

Similarity of quantum and classical mechanics for large quantum numbers

Pair of terms position, linear momentum: probability cloud

Pair of terms wave, particle: wave-particle duality

Probability cloud is switched to common trajectory

Object observed is either wave or particle

Microworld objects are not directly observable
Dirac principle of absolute smallness
Complementarity principle
Uncertainty principle

Classical macroworld objects are directly observable

Correspondence principle

The electron wave-particle duality as basis of new electron theory – electron quantum mechanics.

Results obtained by instrument should be processed mathematically.
How to create a mathematical model of the electron?

Mathematical characteristics of operators

Physical characteristics of observation process

Mathematical model of the electron: Operators instead of quantities, eigenfunctions instead of states, solution of eigenvalue equations of operators (4 quantum numbers for an electron) instead of observation results, Schrödinger equation instead of state change. **How to apply the electron mathematical model?**

Concept of electron

Concept of electron state

Concept of electron motion

Concept of group of electrons

Completion of the "Quantum mechanics of the bound electron" by group of concepts and images concerning the bound electron:

a) Electron image: Wave-particle duality of electron
b) Image of elektron state: Shape of probability cloud determined by 4 quantum numbers
c) Image of elektron motion: Change of probability cloud shape determined by changes of quantum numbers
d) Image of electron group: Overlapping of probability clouds indicating the indistinguishableness of particular electrons

How to apply "Bound electron quantum mechanics"?

Groups of electrons as groups of one type of fermions, statistical physics with numerus applications

Mendeleev system of elements, formation of molecules, quantum chemistry, atomic and nuclear physics with numerous applications

Figure 6: The model of the qunatum non-statistical physics
source: (Záškodný & Procházka, Survey of Principles of Theoretical Physics (with application to radiology), 2014)

In the microworld are fading out corpuscular and wave attributes of particles and wave-particle dualism takes its place – all object in microworld shows attributes of particles as well as waves. Considering matter particles their wave properties are in connection with de Broglie waves. Physical quantities of matter particles are given by mathematical operators – instructions saying what mathematical operations are needed to be done with quantity on the right side from operator. Physical states are determined by complex probability functions. Values of the operators are expressed by set of eigenvalues acquiring discrete values.

Well-known eigenequation is the stationary Schrödinger equation as eigenequation of Hamilton operator which represents energy. The non-stationary Schrödinger equation has got meaning of general quantum and non-relativistic equation of motion which describes changing "probability clouds". (Záškodný, Přehled základů teoretické fyziky (s aplikací na radiologii), 2005) (Záškodný & Procházka, Survey of Principles of Theoretical Physics (with application to radiology), 2014)

Ad 2. – ***intended curriculum***

The intended curriculum requires adapting the knowledge and methods of quantum mechanics to knowledge of students of radiology from mathematics and physics. This adjustment could be expressed by study plan for radiographers at Faculty of Health and Social studies, University of South Bohemia and by the structure of subjects relating to mathematics and physics.

Study plan of **Selected Chapter from Applied Mathematics**:

- System of elementary functions (properties, graphs), Investigation of graph course, limit of a function,
- Elementary algebra (exponentiation, roots, general algebraic modifications, Solving of equations and inequations,
- Differential calculus, Integral calculus,
- Basics of vector calculus. (ZSF JČU)

Study plan of **Radiological physics**:

- Waves, Electromagnetic field,
- Quantum optics, Quantum mechanics of electron,
- Nuclear physics, Sources of ionizing and non-ionizing radiation, Interaction with matter, Detection and dosimetry of radiation and its importance for radiological physics,
- Physics principles of radiologic methods, Physics principles of radiodiagnostics and radiotherapy - summary. (ZSF JČU)

Knowledge acquired in scope of study plan can be already reflected in the description of the projected curriculum.

Ad 3. – ***projected curriculum***

The crucial part of projected curriculum is educational text. On the basis of analysis of communicable scientific system of physics was created educational text “Physical Principles of Magnetic Resonance for Radiographers” which has to comply with the level of physical knowledge of students of radiography and the other related branches of study.

When creating an educational text, the author sought answer to the question: "**How to create a mathematical model of the electron?**" Procedure can be described by several basic steps:

a) „Physical definition of the problem and determination of quantum initial conditions,
b) Delimitation of a complete set of quantities and corresponding operators,
c) Writing and solution of a system of operator eigenvalue equations,
d) Determination of a system of eigenfunctions and system of eigenvalues of operators characterized by quantum numbers,
e) Determination of the shapes of “probability clouds”,
f) Interpretation of results based on the allowable values of quantum numbers.“ (Záškodný & Procházka, Survey of Principles of Theoretical Physics (with application to radiology), 2014)

Consequently, the scientific knowledge was adjusted to the level of excepted recipients.

Ad 4. – ***implemented curriculum*-1**

Implemented curriculum-1 is connected with preparedness of the educator on education. Verification of educational function of the educational text was done by means of experimental lessons. Two forms of education took place:

1. Experimental lecture of the author.
2. Provision of author’s educational text to the students.

The applications for participation were addressed to students of following universities:

- Faculty of Medicine, University of Ostrava,
- Faculty of Health Sciences, Palacký University Olomouc,
- Faculty of Medicine, Masaryk University in Brno,
- Faculty of Health and Social Studies at the University of South Bohemia in Budweis,
- Faculty of Biomedical Engineering, Czech Technical University in Prague
- Faculty of Health Studies, University of West Bohemia in Pilsen,
- Faculty of Health Studies, University of Pardubice,
- The College of Nursing, o.p.s. in Prague.

Both forms of experimental lectures were done only at students at Faculty of Health and Social studies at the University of South Bohemia in Budweis. To the other students, the educational text was just provided.

Ad 5. – ***implemented curriculum-2***

Implemented curiculum-2 studies acquired results by educational process, usually on the basis of educational test and its statistical research. The educational test was outlined as a multiple-choice test with four options. Each question has only one correct answer. The scope of the test was the same as the scope of educational text. The time limit for passing the test was 20 minutes. For each correct answer was given one point, for each wrong nothing happened. The test was fully anonymous. The respondents were only asked for information about the university and grade.

C.4.3. USED STATISTICAL METHODS

Statistical research was done by means mentioned in sources (Záškodný, Havránková, Havránek, & Vurm, 2011), (Zvárová, 2002) and (Záškodný, Základy pravděpodobnosti a statistiky (data miningový přístup), 2013). A few methods of descriptive and mathematical statistics were used. Its list and logical consequence is following:

1) formulation of statistical investigation,
2) creation of scale,
3) measuring in descriptive statistics,
4) elementary statistical processing,
5) non-parametrical testing

C.5. RESULTS

The presented results will be outlined by means of sequence of variant forms of curriculum.

i) Conceptual curriculum

See presented derivation of stationary Schrödinger equation, crucially the role of magnetic quantum number and magnetic spin quantum number as a expression of communicable scientific system in scope of physical principles of magnetic resonance. Further, there is consideration about the non-stationary Schrödinger equation. The derivations can be taken as results mainly for their connection with explaining the physical principles of magnetic resonance for radiographers.

ii) Intended curriculum

The intended curriculum is the adjustment of conceptual curriculum in scope of physical principles of magnetic resonance to capabilities of students of radiography. The insufficiency of the old quantum theory (wave-particle dualism) was shown in this area and the necessity of new quantum theory characterized by operational calculus (see methodology). The model can be taken as result mainly due to its connection with describing physical principles of magnetic resonance for radiographers.

iii) Projected curriculum

Crucial part of projected curriculum is educational text as a output of intended curriculum (see OEDM-SERM 13/14 proceedings).

iv) Implemented curriculum-1

The author's preparedness on experimental lecture was mainly connected with creation of educational test which was expected to reflex acquired knowledge.

v) Implemented curriculum-2

Implemented curriculum-2 reflects results acquired by student of radiography after passing educational process. The results can be quantified mainly with statistical investigation of final score of educational test (see methodology of research).

vi) Formulation of statistical investigation

- **Collective random phenomenon:** measuring knowledge of students of study branch radiography.
- **Statistical unit:** student.
- **Statistical sign:** extent of knowledge.
- **Values of statistical sign:** rate of knowledge expressed by the number of points obtained in the test.
- **Basic statistical set:** 37 students.
- **Selective statistical set:** see basic statistical set.

vii) Creation of scale

It was made 37 measurements. Final scores fits into interval <3;17>. For the purpose of statistical investigation was chosen quantitative metric scale. Individual scales are: (-∞; 7>, <8;9>, <10;11>, <12;13>, <14;15>, <16; +∞).

viii) Elementary statistical processing

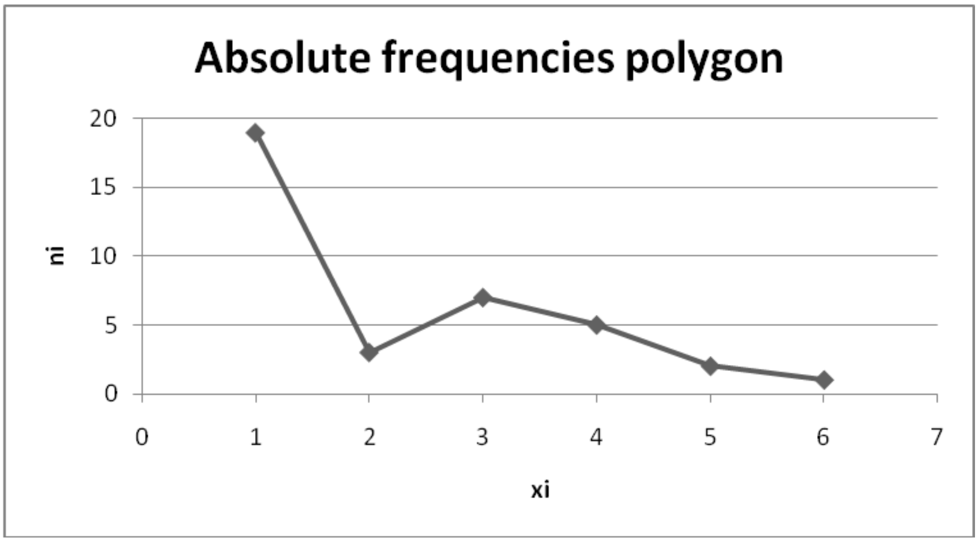
Table 3: Table of elementary statistical processing

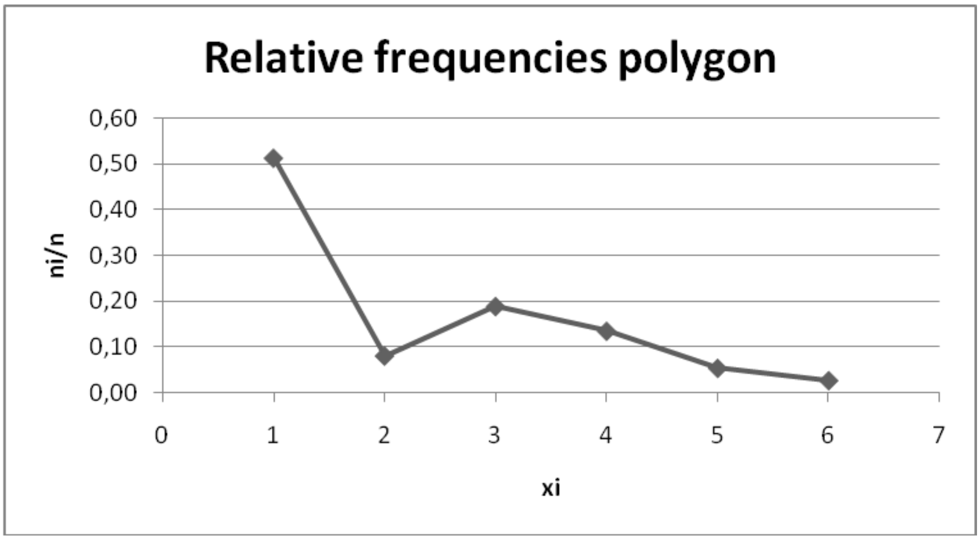
x_i	*mid. int.*	n_i	Σn_i	n_i/n	$\Sigma n_i/n$	$x_i n_i$	$x_i n_i^2$	$x_i n_i^3$	$x_i n_i^4$
(−∞;7>	5	19	19	0,51	0,51	95	475	2375	11875
<8;9>	8,5	3	22	0,08	0,59	25,5	216,75	1842,375	15660,19
<10;11>	10,5	7	29	0,19	0,78	73,5	771,75	8103,375	85085,44
<12;13>	12,5	5	34	0,14	0,92	62,5	781,25	9765,625	122070,3
<14;15>	14,5	2	36	0,05	0,97	29	420,5	6097,25	88410,13
<16;+∞)	17	1	37	0,03	1,00	17	289	4913	83521
Σ		37				302,5	2954,25	33096,63	406622,1

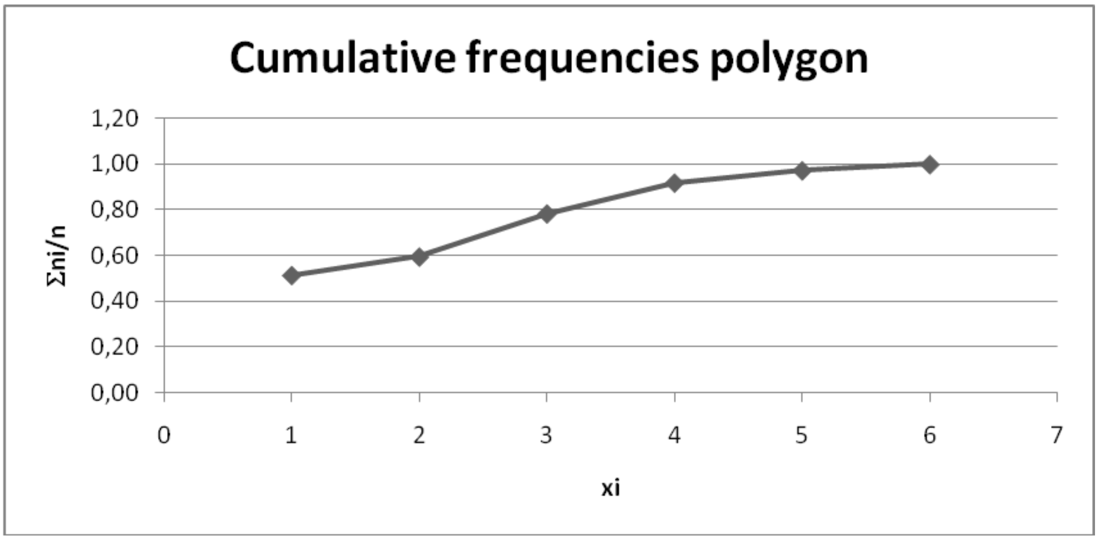
Empirical parameters

- parameter of location
  $O_1 = \bar{x} = 8,175676$
- parameter of variability
  $S_x = 3,605956$
- parameter of obliqueness
  $N_3 = 0,620725$
- parameter of pointendness
  $N_4 = 2,101761$

Empirical frequency







ix) Non-parametrical testing – Pearson χ^2-test distribution

Formula for experimental value of χ^2-test

$$\chi_{\exp}^2 = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i}.$$

Formula for probability function of Poisson distribution is

$$p_i = e^{-\lambda} \frac{\lambda^i}{i!}, i = 0,1,\dots,\infty,$$

λ is theoretical parameter estimated using the 1st general moment O_1. Theoretical parameter of Poisson distribution is $\lambda = O_1 = 8{,}175676$.

- stating zero and alternative hypothesis H_0 a H_a

H_0: Empirical distribution of frequencies is replaceable by Poisson distribution.
H_a: Empirical distribution of frequencies is irreplaceable by Poisson distribution.

- experimental value of χ^2-test $\chi_{exp}{}^2 = 2{,}371349$

- theoretical value of χ^2-test $\chi_{teor}{}^2 = 3{,}84$ on the significance level α = 0,05

- formation of confidence interval on the significance level α = 0,05

$$W \in \langle 3{,}84; +\infty)$$

- acceptance of zero hypothesis H_0, refuse of alternative hypothesis H_a

$$\chi_{\exp}^2 = 2{,}371349 \notin W \in \langle 3{,}84; +\infty)$$

Empirical distribution of frequencies is replaceable by Poisson distribution.

C.6. DISCUSSION

C.6.1. DISCUSSION OF VERIFIED HYPOTHESES

This work was made on the basis of following hypotheses:

1) Educational text for students of the branch of Radiographer can be made by means of application of curricular process.
2) Knowledge of students acquired by means of the elaborated educational text will have a distribution close to normal distribution.

The 1st hypothesis was accepted. The educational text summarizing all physical principles of magnetic resonance was written by means of the curricular process.

From outcomes of statistical investigation can be concluded that the educational text was not adjusted to the level of students study branch Radiographer. One of the hypotheses was: "*Knowledge of students acquired by means of the elaborated educational text will have a* *distribution close to normal distribution.*" and that is not compatible with results of statistical investigation. The knowledge of students shows Poisson theoretical distribution.

The possibility of replacing empirical distribution of frequencies by Poisson distribution was verified by Pearson χ^2-test on the confidence level $\alpha = 0{,}05$. The null hypothesis H_0 was accepted, the alternative hypothesis H_a was refused.

C.6.2. DISCUSSION OF OBTAINED RESULTS OF EDUCATION

The knowledge of invited students was low. Approximately 50% of students acquired final score lower or equal to seven points. On the basis of given outcomes, we can specify hypotheses concerning the reason of such a low final scores. The hypotheses are:

1) Lack of space for theoretical training in field of new quantum theory.
2) Adequacy of the educational text, problems with assembling the educational test.

There are few options that could improve the success.

Ad 1

The educational text was written on the basis of study plan of subjects Radiological Physics and Selected Chapters from Applied Mathematics of study branch Radiographer. Both of them are mandatory from the 1st grade. Curriculum in the educational text included only basic knowledge about functions, operations with them (differential and integral calculus) or algebraic operations with expressions etc.

The educational test, following the experimental education, has to verify acquired knowledge. The test did not require any sophisticated knowledge which comes out from the nature of the multiple-choice test. It would certainly be worth considering whether it would be advantageous to deepen the mathematical and the physical education in the field of new quantum theory. Without operational calculus is difficult to describe physical principles of magnetic resonance.

The results of the test confirm that knowledge of old quantum theory and (mainly) new quantum theory is essential for understanding the physical principles of magnetic resonance. Without setting the full set of quantum numbers is unable to describe the state of electron in atomic orbital. (Záškodný, Přehled základů teoretické fyziky (s aplikací na radiologii), 2005)

$$\{\psi_i\} = \{\psi_{n,l,m_l,m_s}\}$$

Crucially, setting the magnetic quantum number m_l

$$\frac{d^2\Phi}{d\varphi^2} + m_l^2\Phi = 0, \quad m_l \in \mathbb{Z} \wedge m_l \in (-l \ldots 0 \ldots +l)$$

and the magnetic spin quantum number m_s experimentally postulated for example from Stern-Gerlach experiment or Einstein-de Hass experiment. (Janeček, Klaus, & Hřivňák, Kvantová čísla, 2006)

No less significant role for understanding magnetic resonance plays Heisenberg uncertainty principle ($\Delta x \Delta p \approx \hbar$) which is consequences of precession of angular momentum vector L_z. (Beiser, Úvod do moderní fyziky, 1978) (Záškodný, Přehled základů teoretické fyziky (s aplikací na radiologii), 2005)

All in all, it can be pronounced that deeper theoretical knowledge of physics would lay a better basis for indirect master degree study branches as Radiological Physics, Dosimetry of Ionizing Radiation, Biomedical Technique etc. Nevertheless, the presented amount of physical principles was enough to pass the test.

Deeper theoretical basis would provide a potential option of participating in the scientific field, mainly in branches concerning with medicine and physics, only if wanted, of course.

Thorough theoretical training gives possibility of performing various scientific researches. Moreover, it could raise social prestige of this profession. The higher social prestige gives logically the higher interest in the profession.

Ad 2

Processed educational text and educational test reflect contradiction between conceptual and intended curriculum (which require usage of operational calculus at least at simplified from), on the one hand, and projected curriculum and implemented curriculum-1 (which characterize recent concept of bachelor degree study branch Radiographer which has no connection on master degree and so on higher social prestige), on the other hand. Study requirements could be lowered but only to the detriment of decreasing the expertness of students of study branch Radiographer.

C.6.3. DISCUSSION OF RESULTS ACQUIRED IN SCOPE OF APPLICATION OF THE CURRICULAR PROCESS

In scope of conceptual curriculum (communicability of scientific system of physical principles of magnetic resonance) and intended curriculum (educational system of physical principles of magnetic resonance) were acquired outcomes which support necessity of new quantum theory (operational calculus, quantum numbers) for understanding physical principles of magnetic resonance by radiographers.

When analysing recent state of projected curriculum and implemented curriculum-1 which reflect opportunities given by accreditation materials for radiographers was detected important disproportion while creating educational text (projected curriculum) and through realization of implemented curriculum-1 (preparedness for education).

Basically, there was no possibility of including new quantum theory in structure of educational media (mainly educational text). This is then reflected in the evaluation of the results of statistical investigation of the educational text. The Poisson distribution was detected instead of Gauss distribution which shows the disproportion between conceptual and intended curriculum and projected curriculum, implemented currcilum-1 and -2.

It is conceivable that the disproportion could be removed with the introduction of Selected Chapters of General and Theoretical Physics which was included in erstwhile accreditation materials for study branch Radiographer. Otherwise, unsuccessful understanding of physical principles of magnetic resonance leads to descriptive character of this major part of profile of radiographers.

C.7. CONCLUSION

This quantitative research dealt with application of the curricular process (which applicability was accepted by foreign scientific community – see impact publication Záškodný, 2012) of radiological physics in the area of magnetic resonance.

The scientific system of quantum non-statistical physic did undergo *communication transformation* T^1 which outcome is communicable scientific system of quantum non-statistical physics as **conceptual curriculum** (description of observation in quantum physics, derivation of Schrödinger equation, necessity of magnetic quantum number m_l and magnetic spin quantum number m_s for magnetic resonance.

Afterwards, communicable scientific of quantum non-statistical physics did undergo *content transformation* T^2 on didactic system of quantum non-statistical physic and its curriculum (**intended curriculum**) in scope of application on magnetic resonance. The result of content transformation was adjustment of methods and knowledge of quantum non-statistical physics to the level of student of radiography.

Didactic system of quantum non-statistical physics leaded to creation of the educational text (*curricular transformation* T^3) for students of study branch Radiographer which summarizes essential knowledge about magnetic resonance. The creation of educational test (**implemented curriculum-1**) which follows the educational text is logical consequence.

The provision of the educational text to students of radiography (together with experimental education) transformed projected curriculum and implemented curriculum-1 (*educational transformation* T^4) into **implemented curriculum-2** characterized as "the results of education physics principles of magnetic resonance". The measure of knowledge of students of study branch Radiographer was objectively qualified by means of statistical investigation methods.

The first hypothesis "*Educational text for students of the branch of radiographer can be made by means of application of curricular process*." was verified. The educational text is outcome of curricular transformation of intended curriculum.

From results of the statistical research can be deducted that the educational text was not compatible to students of radiography. The hypothesis: "*Knowledge of students acquired by means of the elaborated educational text will have a distribution close to normal distribution.*" was not verified. The outcomes of statistical investigation show that experimental distribution of student's knowledge is close to Poisson distribution with highest absolute frequency in low final scores in the test.

During evaluating the research were stated two additional hypotheses concerning reasons for Poisson distribution:

1) Lack of space for theoretical training in field of new quantum theory.
2) Adequacy of the educational text, problems with assembling the educational test.

Confirmation that it is possible to successfully apply the curricular process in education of students of the branch of radiographers in the area of the bases of magnetic resonance is one of the benefits of the thesis. Success rate of this application was confirmed by means of chosen statistical methods.

Between practical benefits of the thesis it is possible to emphasize a fact that the new quantum theory (operational calculus, full set of quantum number) is a needed physical base for understanding magnetic resonance. Imbalance between the need of new quantum theory and current content of accreditation materials could be removed by implementation of new subject “Selected Chapter of General and Theoretical Physics”.

C.8. SUMMARY OF PART C

Currently, there is not available any simple educational text with the subject "Physics Principles of Magnetic Resonance" which would be adequate to knowledge of students of the study branch of Radiographer and of other related branches of study and which would be based on operational calculus. On the basis of the analysed scientific system of non-statistical quantum physics was made an educational text summarizing all important facts about physical bases of magnetic resonance. Functionality of the educational text was consequently validated by experimental teaching of students of the branch of radiology. Acquired knowledge was verified by creation of an educational test which respected the depth and width of subject matter involved in the educational text. Between practical benefits of the thesis it is possible to emphasize a fact that the new quantum theory (operational calculus, full set of quantum number) is a needed physical base for understanding magnetic resonance. Imbalance between the need of new quantum theory and current content of accreditation materials could be removed by implementation of new subject "Selected Chapter of General and Theoretical Physics".

Presented quantitative research is also issuing from the following sources: Brus (2013), Contemporary Physics Education Project (2006), Fischbein, McGowan, Spencer (2005), Fowler (2014), Geva (2006), James (1998), McGinty (2013), McMurry (2004), Nekula, Chmelova (2004), Rusnak (2010), Tarabek, Cervinkova (2006), Woodward, Freimarck (1995), Vobecky, Zahlava (2001), Valek, Zizka (1996), Zettili (2009).

Keywords

Curricular process, Magnetic resonance, New quantum theory, Operators, Schrödinger equation, Physical bases for Radiographers, Conceptual curriculum, Intended curriculum, Projected curriculum, Implemented curriculum, Quantitative research, Degrees of quantitative research

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D. ASSESSMENT OF SAFEGUARDS TO PROTECT CIVIL AVIATION AGAINST ACTS OF UNLAWFUL INTERFERENCE AT INTERNATIONAL AIRPORTS (ESPECIALLY FOCUSED ON CESKE BUDEJOVICE AIRPORT)

D.1. QUANTITATIVE RESEARCH DEGREES

Presented research “Assessment of Safeguards to Protect Civil Aviation against Acts of Unlawful Interference at International Airports (especially focused on Ceske Budejovice Airport)” can be regarded as close to quantitative research.

The degrees of quantitative research Quantitative research will be briefly characterized through work Záškodný, Záškodná, 2014). The first degree was called “Reporting”, the second ”Exploration”, the third “Explanation”, the fourth “Prediction”.

- The first degree “Reporting” has not a research character and is associated with the characteristics of the simultaneous state of the solved problem investigation. This is achieved by the primary data collection.
- The second degree "Exploration (Description with continuing Classification)" is already a research degree in the form of exploratory research. Within this degree the examined classified phenomenon (as a part of population protection theory in the case of presented quantitative research) is delimited and the pre-scientific and primal hypotheses are taken into account. For this purpose the operations “description” and “classification” are used.
- In the course of the third degree "Explanation" the theoretical hypotheses are articulated and one of them are derived the operating hypotheses with explanatory functions. Based on the operationalization the research variables are delimited and also their measurement. After finalization of measurement process the operating hypotheses can be verified.
- The last of the fourth degree “Prediction” is directed towards the formulation of the research forecasts based on the use of the potential of the hypotheses explanatory functions.

The described four degrees **reporting-exploration-explanation-prediction** can be easily recognized in presented quantitative research.

D.2. INTRODUCTION

The base of the effective safety system in the field of civilian aircraft is quality, motivated and continuously educated employees. This is valid not only for the employees of airports, airline companies and providers of other air services, but also for the policemen who are in charge to service at the international airports and other workers of the Police of the Czech Republic and Czech Ministry of the Interior who are engaged in this field. The term *the safety of* the civilian aircraft can be explained in Czech in two different ways. The safety of the civilian aircraft can be understood as the protection against accidents and other negative events which happen independently on a man or a group of people´s will – safety, or the protection against illegal which means negative acts caused by intentional human acts - „*security.*“ (Urban, 2012).

In this quantitative research report the results which were described in detail in Urban, 2012 are quoted. The aim of the research is to judge the contemporary condition and the level of knowledge regarding the safety of the civilian aircraft at the international airports focusing on the Ceske Budejovice Airport (LKCS). The research is executed via the chosen statistical methods which should prove if they are suitable for this type of research. The results evaluate on a basis of a questionnaire the level of knowledge of the personnel at the Ceske Budejovice and Prague Ruzyne Airport (LKPR) about the safety of the civilian aviation in the field of illegal acts and this level is being compared via the application of the statistical inquiry (the author´s note: in the time when the research was executed the airport was called Ruzyne, today´s name is Vaclav Havel´s Airport).

D.3. ALGORITHM OF STATISTICAL METHODS USED

In presented quantitative research report the following methods were used:

- **Scaling**
- **Measurement**
- **Elementary statistical processing**
- **Empirical division of frequencies**
    - Absolute frequency
    - Relative frequency
- **Parametrical testing**
    - Parameter of the position

- General moments - O_i
- Central moments - C_i
- Normal moments - N_i

- **Paired normal testing**
  - Two-paired t – test
- **Poisson distribution**
  - χ^2 test

D.4. ORIENTATION OF RESEARCH REPORT

The research report tends to comparison of the level of knowledge in the field of security precautions against illegal acts at the international airports in the Czech Republic. In the following paragraphs the methods with which the author has proceeded during the research are given. The algorithm of the mathematical and descriptive statistics and the method of comparison of the results of the structural analyses of the given problems according to the set hypotheses of the research report are dealt with.

D.5. OPERATING HYPOTHESES

The level of the safety at the international airport in Ceske Budejovice corresponds to the standards executed at other international airports in the Czech Republic. These airports correspond from the safety point of view to the valid legislation according to which the safety of the civilian aviation in the Czech Republic follow.

Relative to the definition of the contemporary condition of the research of the solved problem it is possible to state that the level of safety is given by the contemporary legislation which is valid for all international and public international airports this definition enables to formulate the hypothesis H1 in the following way (Urban, 2012).

The formulation of H1 hypothesis:

The knowledge of the workers of the LKCS international airport about the safety is comparable with the knowledge of the workers at the public international airports.

A small modification of the verified hypothesis means that the H1 hypothesis will above all investigate the level of knowledge of the airport´s workers about the safety in the civilian aviation. The statistical inquiry should confirm or disprove the null hypothesis (Urban, 2012).

The formulation of H2 hypothesis:

LKCS international airport corresponds from the safety point of view to the valid legislation which the safety of the civilian aviation in the Czech Republic follows. (Urban, 2012).

D.6. METHODOLOGY

For processing of the research report the international legislation which deals with protective precautions against illegal acts at the international airports is being used. Hereafter the inquiry of airport safety personnel´s knowledge via the questionnaire will follow. The evaluation of these questionnaires follows.

All international public airports must correspond to the safety standards according to the valid international legislation. Questionnaire inquiry is applied at Ceske Budejovice airport and at Prague Ruzyne Airport. The airport was chosen by lots among all international public airports in the Czech Republic. The questionnaires are statistically evaluated and the results are thereafter compared (Urban, 2012).

D.7. METHODOLOGY – H1 STATISTICAL INQUIRY

D.7.1. Questionnaire

The research is based on the application of the questionnaire and following evaluation of the individual questions which are part of the results of the work. The questionnaire is divided into two parts – the first – introductory part questions the rating of the respondent if it is an employee of the aircraft or its user. The other part of the questionnaire includes twelve questions from the field of safety precautions at the international airports in the Czech Republic.

D.7.2. Definition of the statistical inquiry

The methodology H1 is the systematic consequence of the statistical inquiry to compare the level of knowledge of the employees of the two airports: the Prague Ruzyne Airport and the Ceske Budejovice Airport in the form of knowledge questionnaires. The questionnaire inquiry was applied at Prague Ruzyne Airport where 100 questionnaires were distributed to the employees and 68 out of them were replied to. Next the questionnaires were applied at the Ceske Budejovice Airport where 100 questionnaires were given to the employees and 75 of them returned. For the statistical research the chosen statistical set was limited by casting lots and 60 respondents were chosen (Urban, 2012).

D.7.3. Definition of the needful terms of statistical inquiry

The statistical unit „SJ“- is defined by the same qualities of the element of the examined group (people who filled in the questionnaires about the safety precautions against illegal acts in the civilian aviation).

The *statistical indicator „SI“*- is given by some of the different qualities of the element of the researched group (in this case various number of mistakes when filling in the questionnaire is considered).

The value of the statistical indicator „VSI“- is the way the statistical indicator is being described (in this case the range of wrong answers by the respondents 0-5 mistakes is considered).

The basic statistical set BSS“ (population) - is given by all statistical units, its range equals the number of all statistical units (for example the range of the examined basic statistical set BSS is in the beginning of this work the total number of people who are in charge of security at the airport). Usually the statisticians cannot investigate the statistical indicator of all statistical elements and to work with population characteristics. Thus it is suitable to limit the number of statistical units and therefore a fixed number was set SSS 1 and SSS 2 which equals 60.

Random selection „RS“- is the limitation of the number of the investigated statistical units in the way it would be possible to transmit the acquired results on the whole basic statistical set BSS. There are various ways of random sampling (casting lots, generating by the number of random numbers, stratified sampling). It is useful to verify if it is possible to consider the sample as random. The questionnaires in the thesis were chosen by casting lots out of a number of filled and received questionnaires. This form was applied on LKCS and LKPR.

Sample statistical set „SSS“-is connected with sample characteristics and it is given by those statistical units which were chosen from the basic statistical group by the process of random sampling. The range of the sample statistical set equals the number of the chosen statistical units(e.g. the range of the assigned example equals the number of 60 drawn questionnaires). To minimize the sample error as the difference between the population and sample characteristics the range is necessary to be more than 30 statistical units.

D.7.4. Scaling

Scaling is suitable expression of values of the statistical element via *the element of the scale* (grouping of values of the statistical element into suitable groups, the elements of the scale are individual groups). The complex of the elements of the scale is called *the scale*. According to the kind of the statistical element it is possible to distinguish for example four kinds of scales: nominal, ordinal, quantity metric and absolute metric. The classification of scales can be used also for the classification of the statistical elements.

In the practical part of the thesis the values of the statistical element are set by the degrees 1, 2, … 5. The individual elements of the scale are differentiated according to the number of mistakes in the individual questionnaires applied at the international airports. The elements of the scale are the points of the scale expressed by the numbers $x_1 = 1$, $x_2 = 2$, … $x_5 = 5$. This scale should denote the individual number of questionnaires in dependence on the number of incorrect answers (Zaskodny, 2011).

D.7.5. Measuring

Measuring is the process in which every statistical unit of the sample statistical set SSS (of the range of *n* statistical units) is assigned one of *k elements of the scale*
x_1, x_2, …. , x_k. The results of the measurement are detection that the element of the scale x_i (i= 1, 2, …k) was measured n_i times. The total of all values n_i (i= 1, 2, …k), which are called *absolute frequency*, must equal the range *n of the sample statistical set* SSS. In this research report the measurement is executed in the form of analyzing of the results of the questionnaire and following total of the incorrect answers of the individual respondents (Zaskodny, 2011).

D.7.6. Elementary statistical elaboration – the chart

The chart introduces the form of arrangement of the results of the measurement. The chart contains 8 columns. The first four columns are useful not only for the results to be well-arranged (the task – arrangement), but also for the representation of the empirical distributions (the task- graphic expression). The remaining four columns serve as a help for easy and fast calculation of the empirical parameters (the task-parameters). (Zaskodny, 2011).

The first four columns contain:

1.The column indicated x_i - the elements of the scale,

2.The column indicated n_i - absolute frequency of the elements of the scale, 3.The column indicated $\frac{n_i}{n}$ - relative frequency of the elements of the scale,

4.The column indicated $\sum \frac{n_i}{n}$ - cumulative frequency.

Other four columns contain products useful for the calculation of the empirical parameters:

5.Column contains products $x_i \cdot n_i$,

6.Column contains products $x_i^2 \cdot n_i$,

7.Column contains products $x_i^3 \cdot n_i$,

8.Column contains products $x_i^4 \cdot n_i$,

D.7.7. Empirical distributions of frequencies

The empirical distributions of frequencies can be divided into two basic kinds. The first kind assigns the elements of the scale x_i the corresponding absolute frequencies n_i or relative frequencies $\frac{n_i}{n}$. The second kind assigns the elements of the scale x_i the corresponding cumulative frequencies $\sum \frac{n_i}{n}$. The graphic expression of the empirical distribution of the one-dimensional statistical set is linked with using of the coordinate system in the plane. In this coordinate system the elements of the scale belong to the horizontal axis and the corresponding frequencies belong to the vertical one. The graphic expression of these functional dependencies is given by the set of points the coordinate of which is always the element of the scale x_i, the second coordinate is the corresponding frequency. After connecting the neighbouring points of this set by the abscissas it is possible to obtain a pointed line which is called the polygon. It is possible to distinguish the polygon of absolute frequencies, the polygon of relative frequencies, the polygon of cumulative frequencies.

The graphic expression enables the immediate research which the theoretical distribution (from the point of view of the theory of probability) nears to the empirical distribution received as the result of the empirical statistics. The next importance grounds in an immediate orientation evaluation of the parameters of location, variability, obliqueness and sharpness of the empirical distribution (Zaskodny, 2011).

D.7.8. General relations for empirical parameters

The location parameter O_1 (the general moment of 1st order) shows the location of the empirical distribution on the horizontal axis

- The general moment of r-order: $O_1(x) = \bar{x}$ (arithmetic mean)
- The central moment of r- order: $C_r(x) = \frac{1}{n}\sum n_i (x_i - \bar{x})^r$
- The central moment of 2nd order: $C_r(x) = S_x^2$ (empirical dispersion)

The standard deviation (S_x), expressed by square root of C_2, informs about the notice value of the arithmetic mean.

- *The standard deviation:* $S_x = \sqrt{C_2(x)}$

The expression of the necessary central moments with the help of general moments

- $C_2(x) = O_2(x) - [O_1(x)]^2$
- $C_3(x) = O_3(x) - 3O_2(x) \cdot O_1(x) + 2[O_1(x)]^3$
- $C_4(x) = O_4(x) - 4 \cdot O_3(x) \cdot O_1(x) + 6 \cdot O_2(x) \cdot [O_1(x)]^2 - 3 \cdot [O_1(x)]^4$

The expression of the necessary standardized moments with the help of the central moments

- $N_3(x) = \frac{C_3(x)}{C_2(x) \cdot \sqrt{C_2(x)}}$
- $N_4(x) = \frac{C_4(x)}{[C_2(x)]^2}$

D.7.9. Two-sample t – test

After the realization of the empirical inquiry and after the calculation of the important values necessary for other statistical calculation two-sample t-test is applied the task of which is whether the knowledge of the employees of LKCS and LKPR is approximately the same (it is possible to accept the null hypothesis H_0) or if there are differences between the individual statistical indicators of LKCS and LKPR (the alternative hypothesis H_a).

The expression of the general equation for the calculation of t – test

- $$t_{exp} = \frac{x - y}{\sqrt{(n_1 - 1)Sx^2 + (n_2 - 1)Sy^2}} \cdot \sqrt{\frac{n_1 n_2 (n_1 - n_2 - 2)}{n_1 + n_2}}$$
- $$W \in \left(-\infty, -t_{n1+n2-2}\left(\frac{\alpha}{2}\right)\right\rangle \cup \left\langle t_{n1+n2-2}\left(\frac{\alpha}{2}\right), \infty\right)$$
- $x = \mu_1$ » *LKCS*, $y = \mu_2$ » *LKPR*

D.7.10. Poisson distribution

After the calculation of two-sample t test we can test the results of the statistical inquiry if they correspond with the Poisson distribution. The Poisson distribution is used to approximation of the binomial distribution for a large number of experiments, that means for. $n \to \infty$ a little probability of the incidence of the studied phenomenon in one experiment that means $p \to 0$. Usually we can approximate the binomial distribution to the Poison if $n > 30$.

- After the calculation of the general moment of 1. order (O_1) the areas of Poisson distribution are calculated (41, 50):
- $O_1 = \lambda$
- $$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$
- Next Pearson´s chi-squared test χ^2 is applied
- $$\chi^2_{exp} = \sum \frac{(n_i - nP_i)^2}{nP_i}$$
- For the calculation *the robust analysis* will be used
- The critical value χ^2 is chosen according to the statistical tables on the importance level 0,025 with 3 degrees of discretion: 9,35.

D.8. METHODOLOGY - H2 STATISTICAL INQUIRY

- The structural analyses of the legislative regarding the safety of the civilian aviation
- The structural analyses of the legislative system regarding the safety at the international airport LKCS.

The comparison of the structures of both defined system – the aim is to find out the differences (comparison of the international safety program and the safety program of the airport LKCS) (Urban, 2012).

D.9. RESULTS - H1 VERIFICATION

D.9.1. Analysis of questionnaire individual questions

Airport marking:

PRAHA RUZYNE – LKPR

CESKE BUDEJOVICE – LKCS

The chart 1 – The number of correct answers – the question No. 1

The airports	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	65	86,67
LKPR	63	92,65

Source: Personal calculation

- The question No.1 was answered correctly by:
- 65 (86,67%) respondents out of 75 (100%) from the airport LKCS
- 63 (92,65%) respondents out of 68 (100%) from the airport LKPR

1. The public area of the airport is:

x The starting runway of the airport (RWY).

✓ The public area of the airport determined by the entrepreneur, which is not closed part of the airport and includes all areas of the airport open to the public

x Non public area of the airport determined by the entrepreneur, which is not closed part of the airport and includes all areas of the airport open to the public.

The chart 2 – The number of correct answers– the question No. 2

THE AIRPORT	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	74	98,67
LKPR	66	97,06

Source: Personal calculation

- The question No. 2 was answered correctly by:
- 74 (98,67%) respondents out of 75 (100%) from the airport LKCS
- 66 (97,06%) respondents out of 68 (100%) from the airport LKPR

2. The non public area of the airport is:

✓ Non public part of the airport determined by the entrepreneur consisting from locomotor and checking in area, adjacent terrain and buildings or their parts, to which the access is checked.
x Public part of the airport determined by the entrepreneur, which consists of all areas open to public.
x Starting runway of the airport (RWY).

The chart 3 –The number of correct answers – question No. 3

THE AIRPORTS	CORRECT ANSWERS	
	n_i	P_i(%)
LKCS	74	98,67
LKPR	67	97,53

Source: Personal calculation

- The question No. 3 was answered correctly by:
- 74 (98,67%) respondents out of 75 (100%) at the airport LKCS
- 67 (98,53%) respondents out of 68 (100%) at the airport LKPR

3. The safety precautions are:

x Only the precautions of the entrepreneur for the safety of the landing airplanes.
✓ The combination of precautions, human and material means for the civil air traffic to be used to be protected against illegal acts.
x The precautions of the pilot to guarantee a safe flight.

The chart 4 – The number of correct answers– the question No. 4

THE AIRPORTS	CORRECT ANSWERS	
	n_i	P_i(%)
LKCS	75	100,00
LKPR	67	97,53

Source: Personal calculation

- The question No. 4 was answered correctly by:
- 75(100%) respondents out of 75 (100%) at the airport LKCS
- 67 (98,53%) respondents out of 68 (100%) at the airport LKPR

4. The safety check encompasses:

x Only check in of the luggage.
✓ A set of precautions including the detection check and physical check to prevent the weapons, explosives and other dangerous objects and substances to be used to commit an illegal act.
x Only the check of the foreign users of the airport.

The chart 5 – The number of correct answers– the question No. 5

THE AIRPORTS	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	74	98,67
LKPR	68	100,00

Source: Personal calculation

- The question No. 5 was answered correctly by:
- 74 (98,67%) respondents out of 75 (100%) at the airport LKCS
- 68 (100%) respondents out of 68 (100%) at the airport LKPR

5. The hand-held or to put it differently the physical check is necessary in these cases:

x If the passenger brings about the warning signal when passing through the detector of metals.
x During the risk situations.
✓ In both cases given above.

The chart 6 – The number of correct answers– the question No. 6

THE AIRPORTS	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	72	96,00
LKPR	66	97,06

Source: Personal calculation

- The question No. 6 was answered correctly by:
- 72 (96,00%) respondents out of 75 (100%) at the airport LKCS
- 66 (97,06%) respondents out of 68 (100%) at the airport LKPR

6. An illegal act (in the context of air traffic) is:

✓ The action which can have adverse consequences for the safety of the civilian air traffic.
x The action which can have auspicious consequences for the safety of the civilian air traffic.
x It does not essentially influence the course of the air traffic.

The chart 7 – The number of correct answers– the question No. 7

THE AIRPORTS	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	74	98,67
LKPR	68	100,00

Source: Personal calculation

- The question No. 7 was answered correctly by:
- 74 (98,67%) respondents out of 75 (100%) at the airport LKCS
- 68 (100%) respondents out of 68 (100%) at the airport LKPR

7. What are the requirements to enter the non public area of the airport:

x The entry is allowed only to the employees of the airport.
✓ The entry is allowed only to the employees, contractual users and people who have a valid identification certificate.
x The entry to non public area is free for everybody.

The chart 8 – The number of correct answers– the question No. 8

THE AIRPORTS	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	73	97,33
LKPR	67	98,53

Source: Personal calculation

- The question No. 8 was answered correctly:
- 73 (97,33%) respondents out of 75 (100%) at the airport LKCS
- 67 (98,53%) respondents out of 68 (100%) at the airport LKPR

8. What are the requirements to enter the operating areas of the airport:

- x The entry is allowed only to the air personnel.
- ✓ The entry is executed on the basis of the permission of the service of the air traffic control.
- x The entry does not come under any permission.

The chart 9 – The number of correct answers – the question No. 9

THE AIRPORT	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	74	98,67
LKPR	67	98,53

Source: Personal calculation

- The question No. 9 was answered correctly by:
- 74 (98,67%) respondents out of 75 (100%) at the airport LKCS
- 67 (98,53%) respondents out of 68 (100%) at the airport LKPR

9. Which regulations are in connection with the safety precautions practised at the airport?

- ✓ REGULATION L-17 and its supplements regarding the National Safety Program
- x AIP – (airborne information handbook)
- x LAA 1

The chart 10 – The number of correct answers– the question No. 10

AIRPORT	CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	70	93,33
LKPR	65	95,59

Source: Personal calculation

- The question No. 10 was answered correctly by:
- 70 (93,33%) respondents out of 75 (100%) at the airport LKCS
- 65 (95,59%) respondents out of 68 (100%) at the airport LKPR

10. What is the period of validity of the INTEGRATED safety training?

- x 1 year
- ✓ 2 years
- x 3 years

The chart 11 – The number of correct answers – the question No. 11

THE AIRPORT	THE CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	71	94,67
LKPR	65	95,59

Source: Personal calculation

- The question No. 11 was answered correctly:
- 71 (94,67%) respondents out of 75 (100%) at the airport LKCS
- 65 (95,59%) respondents out of 68 (100%) at the airport LKPR

11. The airport identification card must be visibly worn:

✓ In the non public part of the airport.
x In public part of the airport.
x Do not have to be visibly worn in any part of the airport area.

The chart 12 – The number of correct answers– the question No. 12

THE AIRPORT	THE CORRECT ANSWERS	
	n_i	$P_i(\%)$
LKCS	75	100,00
LKPR	68	100,00

Source: Personal calculation

- The question No. 12 was answered correctly by:
- 75 (100%) respondents out of 75 (100%) at the airport LKCS
- 68 (100%) respondents out of 68 (100%) at the airport LKPR

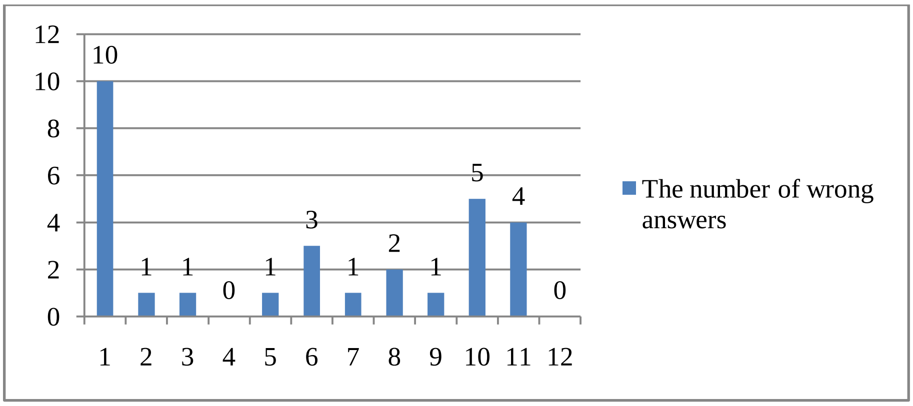
12. Question No.12

Due to 100% result the correct and incorrect possibilities will not be presented

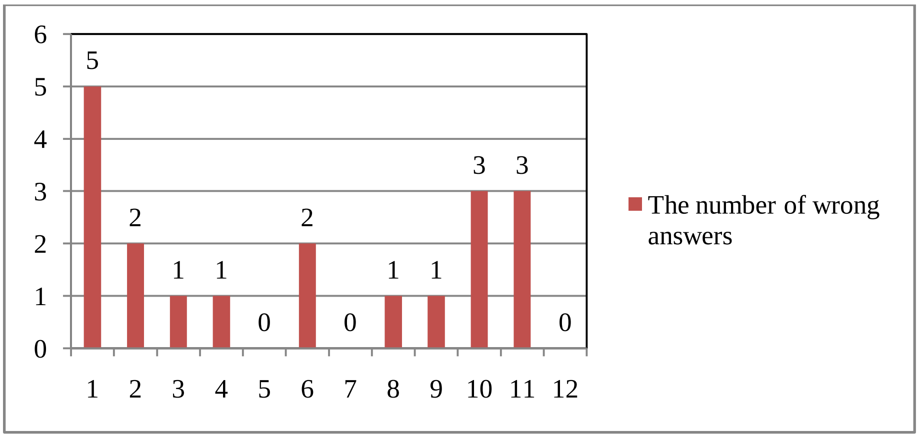
The analysis of questionnaire individual questions is finalized

D.9.2. Graphic demonstration of questionnaire inquiry (Urban, 2012)

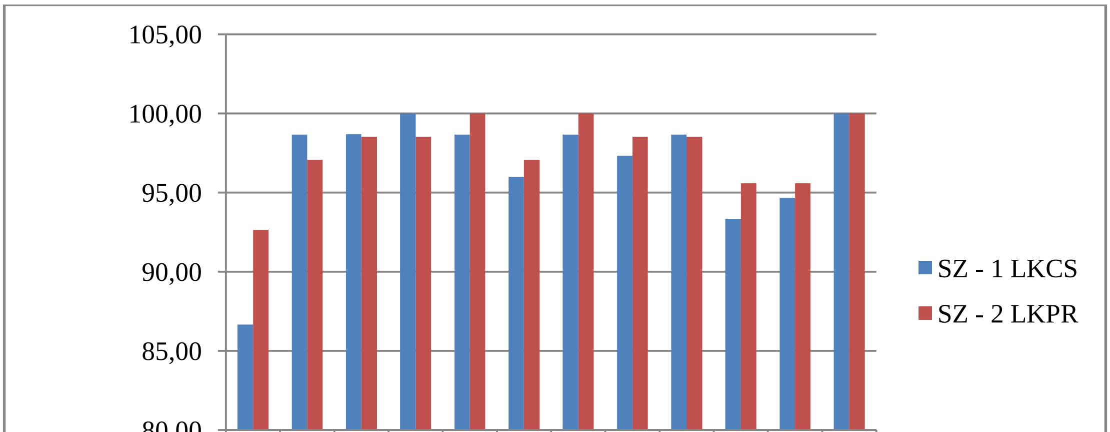
The graph 1 – The number of wrong answers LKCS



The graph 2 – The number of wrong answers LKPR



The graph 3 – The percentage comparison of the results of LKCS and LKPR



	1	2	3	4	5	6	7	8	9	10	11	12
SZ - 1 LKCS	86,	98,	98,	100	98,	96	98,	97,	98,	93,	94,	100
SZ - 2 LKPR	92,	97,	98,	98,	100	97,	100	98,	98,	95,	95,	100

Source: Personal calculation

D.9.3. Elementary statistical processing (Urban, 2012)

The legend of the abbreviations needed to carry out the statistical calculation
HNJ 1 – accumulative event Ceske Budejovice airport (60 questionnaires out of 75 was drawn for the statistical inquiry LKCS)
HNJ 2 – accumulative event Praha Ruzyne airport (60 questionnaires out of 68 was drawn for the statistical inquiry LKPR)
SJ 1 – the statistical unit (the questionnaires filled for LKCS)
SJ 2 – the statistical unit (the questionnaires filled for LKPR)
SZ 1 – the statistical indicator (wrong answers of the questionnaires applied on LKCS)
SZ 2 – the statistical indicator (wrong answers of the questionnaires applied on LKPR)
HSZ 1 - the value of the statistical indicator SZ1 (the number of wrong answers of the questionnaire at LKCS)
HSZ 2 – the value of the statistical indicator SZ2 (the number of wrong answers of the questionnaire at LKPR)
BSS – the basic statistical unit (the collection of all filled in and returned questionnaires at LKCS and LKPR)
NH 1 - random selection **(**60 questionnaires out of 75 was drawn for the statistical inquiry)
NH 2 – random selection (60 questionnaires out of 68 returned was drawn for the statistical inquiry)
SSS 1,2 - *sample statistical set* – LKCS, LKPR

Scaling
SSS 1 = The number of wrong answers LKCS
SSS 2 = The number of wrong anwers LKPR

The chart 13 – the scaling SSS 1 and SSS 2

SSS 1 – LKCS		SSS 2 – LKPR	
x_i	n_i	x_i	n_i
1	46	1	50
2	6	2	4
3	4	3	4
4	3	4	2
5	1	5	0
	n =60		**n =60**

Source: Personal calculation

D.9.3.1. The charts for SSS 1 and SSS 2

The chart 14 – construction SSS 1 for LKCS

	Absol. freq.	*Relative frequency*	*Cumul. frequency*				
x_i	n_i	$\frac{n_i}{n}$	$\sum n_i/n$	$x_i \cdot n_i$	$x_i^2 \cdot n_i$	$x_i^3 \cdot n_i$	$x_i^4 \cdot n_i$
1	46	0,76	0,76	46	46	46	46
2	6	0,1	0,86	12	24	48	96
3	4	0,06	0,93	12	36	108	324
4	3	0,05	0,98	12	48	192	768
5	1	0,02	1	5	25	125	625
	60	**1**		**87**	**179**	**519**	**1859**

Source: Personal calculation

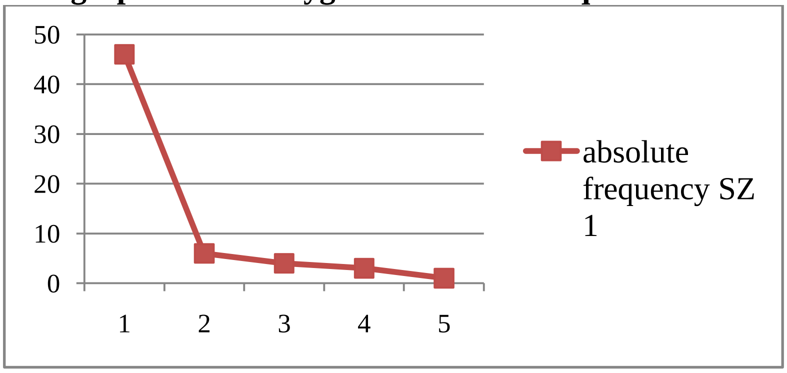
The chart 15 – construction SSS 2 for LKPR

	Absol. freq.	*Relative frequency*	*Cumul frequency*				
x_i	n_i	$\frac{n_i}{n}$	$\sum n_i/n$	$x_i \cdot n_i$	$x_i^2 \cdot n_i$	$x_i^3 \cdot n_i$	$x_i^4 \cdot n_i$
1	50	0,83	0,83	50	50	50	50
2	4	0,06	0,9	8	16	32	64
3	4	0,06	0,96	12	36	108	162
4	2	0,03	1	8	32	128	512
5	0	0	1	0	0	0	0
	60	**1**		**78**	**134**	**318**	**788**

Source: Personal calculation

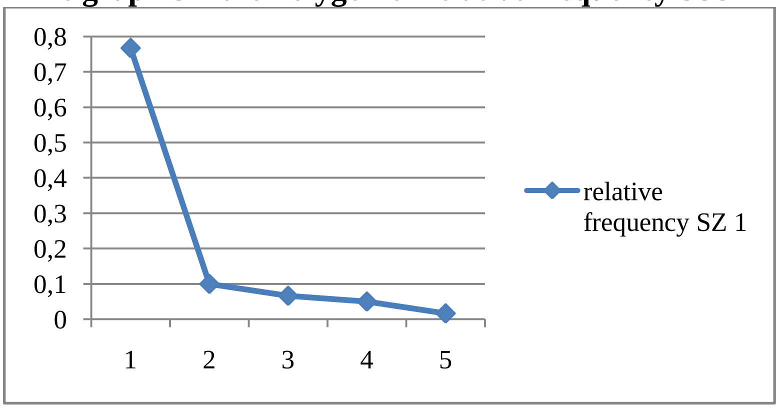
D.9.3.2. The graphs for SSS 1

The graph 4 – the Polygon of absolute frequencies SSS 1



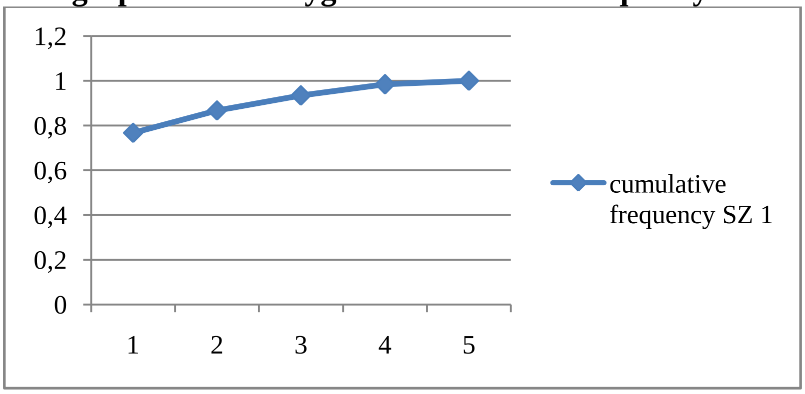
Source: Personal calculation

The graph 5 – the Polygon of relative frequency SSS 1



Source: Personal calculation

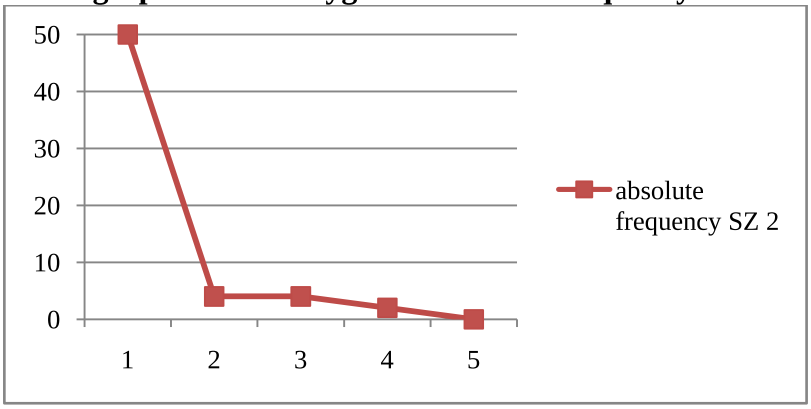
The graph 6 – the Polygon of cumulative frequency SSS 1



Source: Personal calculation

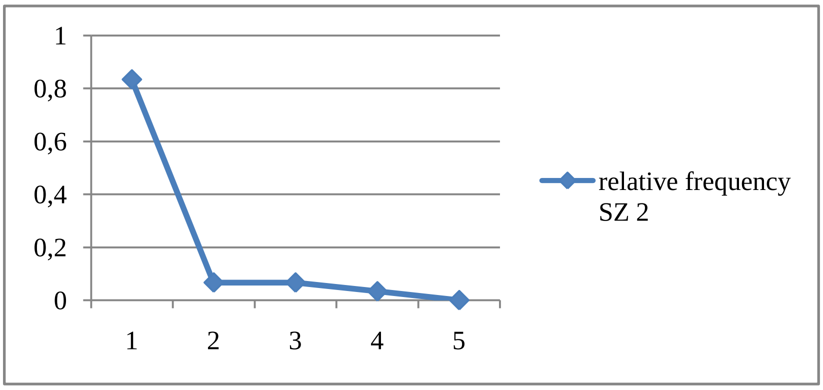
D.9.3.3. The graphs for SSS 2

The graph 7 – the Polygon of absolute frequency SSS 2



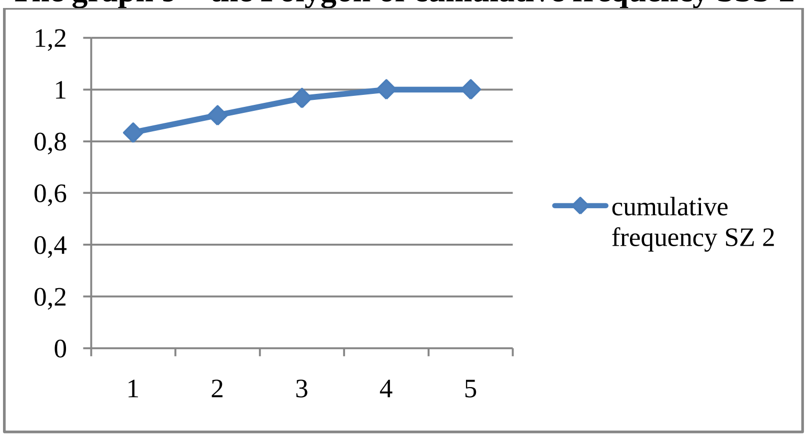
Source: Personal calculation

The graph 8 – the Polygon of relative frequency SSS 2



Source: Personal calculation

The graph 9 – the Polygon of cumulative frequency SSS 2



Source: Personal calculation

D.9.3.4. The calculation of empirical parameters for SSS 1 and SSS 2 (Urban, 2012)

The chart 16 – SSS 1 (LKCS)

	Absolute frequency	*Relative frequency*	*Cumul. frequency*				
x_i	n_i	$\frac{n_i}{n}$	$\sum n_i/n$	$x_i \cdot n_i$	$x_i^2 \cdot n_i$	$x_i^3 \cdot n_i$	$x_i^4 \cdot n_i$
1	46	0,76	0,76	46	46	46	46
2	6	0,1	0,86	12	24	48	96
3	4	0,06	0,93	12	36	108	324
4	3	0,05	0,98	12	48	192	768
5	1	0,01	1	5	25	125	625
	60	**1**		**87**	**179**	**519**	**1859**

Source: Personal calculation

$$O_1 = \sum \frac{x_i \cdot n_i}{n}$$
$$O_1 = \sum \frac{87}{60}$$
$$O_1 = 1{,}45$$

$$O_3 = \sum \frac{x_i^3 n_i}{n}$$
$$O_3 = \sum \frac{519}{60}$$
$$O_3 = 8{,}65$$

$$S_x = \sqrt{C_2}$$
$$S_x = \sqrt{1{,}29}$$
$$S_x = 1{,}14$$

$$O_2 = \sum \frac{x_i^2 \cdot n_i}{n}$$
$$O_2 = \sum \frac{179}{60}$$
$$O_2 = 2{,}98$$

$$O_4 = \sum \frac{x_i^4 \cdot n_i}{n}$$
$$O_4 = \sum \frac{1859}{60}$$
$$O_4 = 30{,}98$$

$$C_2 = O_2 - O_1^2$$
$$C_2 = 2{,}98 - 1{,}3^2$$
$$C_2 = 1{,}29$$

$$C_3(x) = O_3(x) - 3O_2(x) \cdot O_1(x) + 2[O_1(x)]^3$$
$$C_3 = 8{,}56 - 3 \cdot 2{,}98 \cdot 1{,}3 + 2 \cdot (1{,}3)^3$$
$$C_3 = 1{,}42$$

$$C_4(x) = O_4(x) - 4 \cdot O_3(x) \cdot O_1(x) + 6 \cdot O_2(x) \cdot [O_1(x)]^2 - 3 \cdot [O_1(x)]^4$$
$$C_4 = 4{,}65$$

$$N_3 = \frac{C_3(x)}{C_2(x) \cdot \sqrt{C_2(x)}}$$
$$N_3 = \frac{1{,}42}{1{,}29 \cdot \sqrt{1{,}29}}$$
$$N_3 = 1{,}24$$

$$N_4 = \frac{C_4(x)}{[C_2(x)]^2}$$
$$N_4 = \frac{7{,}56}{(1{,}29)^2}$$
$$N_4 = 4{,}60$$

The chart 17 – SSS 2 (LKPR)

	Absolute freq.	Relative frequency	Cumulative frequency				
x_i	n_i	$\frac{n_i}{n}$	$\sum n_i/n$	$x_i \cdot n_i$	$x_i^2 \cdot n_i$	$x_i^3 \cdot n_i$	$x_i^4 \cdot n_i$
1	50	0,83	0,83	50	50	50	50
2	4	0,06	0,9	8	16	32	64
3	4	0,06	0,96	12	36	108	162
4	2	0,03	1	8	32	128	512
5	0	0	1	0	0	0	0
	60	**1**		**78**	**134**	**318**	**788**

Source: Personal calculation

$$O_1 = \sum \frac{x_i \cdot n_i}{n} \quad O_1 = \sum \frac{78}{60} \quad O_1 = 1.3$$

$$O_2 = \sum \frac{x_i^2 \cdot n_i}{n} \quad O_2 = \sum \frac{134}{60} \quad O_2 = 2.33$$

$$O_3 = \sum \frac{x_i^3 \cdot n_i}{n} \quad O_3 = \sum \frac{318}{60} \quad O_3 = 5.3$$

$$O_4 = \sum \frac{x_i^4 \cdot n_i}{n} \quad O_4 = \sum \frac{788}{60} \quad O_4 = 13.13$$

$$C_2 = O_2 - O_1^2$$

$$C_2 = 2,33 - 1,3^2$$

$$C_2 = 0,64 \quad S_x = \sqrt{0.64} = 0.8$$

$$C_3 = 0.61$$

$$C_4 = 0.63$$

$$N_3 = 0.76$$

$$N_4 = 1.54$$

D.9.4. Parametrical testing

The statistical criteria: two-sample t – test

$$t_{\exp} = \frac{x - y}{\sqrt{(n_1 - 1)Sx^2 + (n_2 - 1)Sy^2}} \cdot \sqrt{\frac{n_1 n_2 (n_1 - n_2 - 2)}{n_1 + n_2}}$$

$$W \in \left(-\infty, -t_{n1+n2-2}\left(\frac{\alpha}{2}\right)\right\rangle \cup \left\langle t_{n1+n2-2}\left(\frac{\alpha}{2}\right), \infty\right)$$

$x = \mu_1$ » LKCS, $y = \mu_2$ » LKPR

The formulation of null and alternative hypothesis for H1:

H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$

The statistical criteria:

$$t_{\exp} = \frac{x - y}{\sqrt{(n_1 - 1)Sx^2 + (n_2 - 1)Sy^2}} \cdot \sqrt{\frac{n_1 n_2 (n_1 - n_2 - 2)}{n_1 + n_2}}$$

The critical values:

$$-t_{n1+n2-2}\left(\frac{\alpha}{2}\right), t_{n1+n2-2}\left(\frac{\alpha}{2}\right)$$

The critical region:

$$W \in \left(-\infty, -t_{n1+n2-2}\left(\frac{\alpha}{2}\right)\right\rangle \cup \left\langle t_{n1+n2-2}\left(\frac{\alpha}{2}\right), \infty\right)$$

$$T_{1,2}(0{,}025) = 1{,}96 \Rightarrow W = (-\infty; -1.96\rangle \cup \langle 1.96; \infty)$$

The values calculated from the previous statistical inquiry:

$S_x = 1{,}14$ $\quad$ $n_1 = 60$

$S_y = 0{,}8$ $\quad$ $n_2 = 60$

$O_1 = x = 1{,}45$ $\quad$ $O_1 = y = 1{,}3$

After the substitution:

$$t_{\exp} = \frac{x - y}{\sqrt{(n_1 - 1)Sx^2 + (n_2 - 1)Sy^2}} \cdot \sqrt{\frac{n_1 n_2 (n_1 - n_2 - 2)}{n_1 + n_2}}$$

$$t_{\exp} = 0{,}83 \Rightarrow t_{\exp} \neq W \Rightarrow H_0$$

The interpretation of result

The experimental value t_{exp} does not belong to the critical region on the level of importance α = 0.025, the null hypothesis H_0 can be accepted.

The employees of the airports LKCS and LKPR have on the level of importance α = 0.025 approximately same knowledge about the safety in the civilian aviation in the Czech Republic (Urban, 2012).

D.9.5. Non-parametrical testing

The Poisson distribution

	Absolute frequency	*Relative frequency*
x_i	n_i	$\frac{n_i}{n}$
1	46	0,76
2	6	0,1
3	4	0,06
4	3	0,05
5	1	0,016
	60	**Σ 1**

	Absolute frequency	*Relative frequency*
x_i	n_i	$\frac{n_i}{n}$
1	50	0,83
2	4	0,06
3	4	0,06
4	2	0,03
5	0	0
	60	**Σ 1**

The calculation for SSS 1:

$$O_1 = \sum 0 + 0,1 + 0,132 + 0,15 + 0,064$$

$$O_1 = 0,446 = \lambda$$

$P_0 = 0,64$, $P_1 = 0,285$, $P_2 = 0,063$, $P_3 = 0,0094$, $P_4 = 0,0011$

$$\chi^2_{\exp} = \sum \frac{(n_i - nP_i)^2}{nP_i}$$

$$\chi^2_{\exp} = \sum 1,504 + 7,205 + 0,012$$

$$\chi^2_{\exp} = 8,721$$

Critical region $\chi_{k-r-1}{}^2(0,025) = \chi_{5-1-1}(0,025) = \chi_3{}^2(0,025) = 9,35$, W=‹9,35, ∞)

$\chi^2_{exp} \leq \chi^2_{krit}$

The interpretation of the result for SSS 1: As $\chi_{exp}{}^2$ does not belong to the critical region W, the empirical distribution of the results of the statistical inquiry of VSS - 1 can be substituted by the discrete theoretical distribution – the Poisson distribution (Urban, 2012).

The calculation for SSS 2:

$$O_1 = \sum 0 + 0{,}066 + 0{,}132 + 0{,}099 + 0$$

$$O_1 = 0{,}297 = \lambda$$

$P_0 = 0{,}743$, $P_1 = 0{,}22$, $P_2 = 0{,}032$, $P_3 = 0{,}0032$, $P_4 = 0{,}00024$

$$\chi^2_{\exp} = \sum \frac{(n_i - nP_i)^2}{nP_i}$$

$$\chi^2_{\exp} = \sum 0{,}64 + 6{,}441 + 2{,}25 + 0{,}014$$

$$\chi^2_{\exp} = 9{,}32$$

The critical region $\chi_{k\text{-}r\text{-}1}{}^2\,(0{,}025) = \chi_{5\text{-}1\text{-}1}\,(0{,}025) = \chi_3{}^2\,(0{,}025) = 9{,}35$ W=‹9,35, ∞)

$\chi^2_{exp} \leq \chi^2_{krit}$

The interpretation of the result for SSS 2**:** As $\chi_{exp}{}^2$ does not belong to the critical region W, the empirical distribution of the results of the statistical inquiry of SSS 2 can be substituted by the discrete theoretical distribution – the Poisson distribution (Urban, 2012).

D.9.6. Paired t – test (Poisson distribution)

Critical region:

$$W \in \left(-\infty, -t_{n_1+n_2-2}\left(\frac{\alpha}{2}\right)\right\rangle \cup \left\langle t_{n_1+n_2-2}\left(\frac{\alpha}{2}\right), \infty\right)$$

$$t_{1,2}\,(0{,}025) = 1{,}96 \Rightarrow W = (-\infty; -1.96\rangle \cup \langle 1.96; \infty)$$

The values for the calculation of t – test:

SSS 1

$$\mu_1 = O_1 = \frac{\chi^2_{\exp}}{5}$$

$$\mu_1 = O_1 = 1{,}74$$

$$C_2 = \sum \frac{n_{ix}}{n \cdot (\chi - O_1)^2}$$

$$C_2 = 9{,}27$$

$$Sx_{čb} = \sqrt{C_2}$$

$$Sx = 3{,}03$$

SSS 2

$$\mu_2 = O_1 = \frac{\chi^2_{\exp}}{5}$$

$$\mu_2 = O_1 = 1{,}86$$

$$C_2 = \sum \frac{n_{ix}}{n \cdot (\chi - O_1)^2}$$

$$C_2 = 10{,}1$$

$$Sy_{pr}\sqrt{C_2}$$

$$Sy = 3{,}18$$

The calculation of t – test of the Poisson distribution:

$$t_{\exp} = \frac{\mu_1 - \mu_2}{\sqrt{(n_1 - 1)Sx^2 + (n_2 - 1)Sy^2}} \cdot \sqrt{\frac{n_1 n_2 (n_1 - n_2 - 2)}{n_1 + n_2}}$$

After the substitution:

$$t_{\exp} = -0{,}21$$

$$t_{\exp} = -0{,}21 \Rightarrow t_{\exp} \neq W \Rightarrow H_0$$

The interpretation of the result:

The experimental value t_{exp} in Poisson distribution does not belong to the critical region on the level of importance α = 0,025, the null hypotheses H_0 can be accepted. The employers of the airports LKCS and LKPR have on the level of importance α = 0,025 approximately same knowledge about the safety in the civilian aviation in the Czech Republic (Urban, 2012).

D.10. RESULTS – H2 VERIFICATION (Urban, 2012)

All international airports in the Czech Republic follow the valid international and internal directives which together with the safety apparatus guarantee the safety of the civilian aviation against illegal acts.

- In the first part of the results H2 there are legislative regulations regarding the safety at the international airports in the Czech Republic arranged according to the structure. There are also described the subjects which also have major influence on the execution of safety and which are responsible regarding the safety of the civilian aviation.
- In the second part of the results H2 there are described the legislative regulations relating specifically the Ceske Budejovice airport.
- The third part of the results H2 is the comparison of both previous analyses.
-

The structural analyses of the system of the legislation relating the safety of the civilian aviation

The analysis connects the theoretical part where the system of the legislation has been itemized and explained, after the execution of the analyses in the field of the safety of the civilian aviation on the state level there are used particularly these regulations:

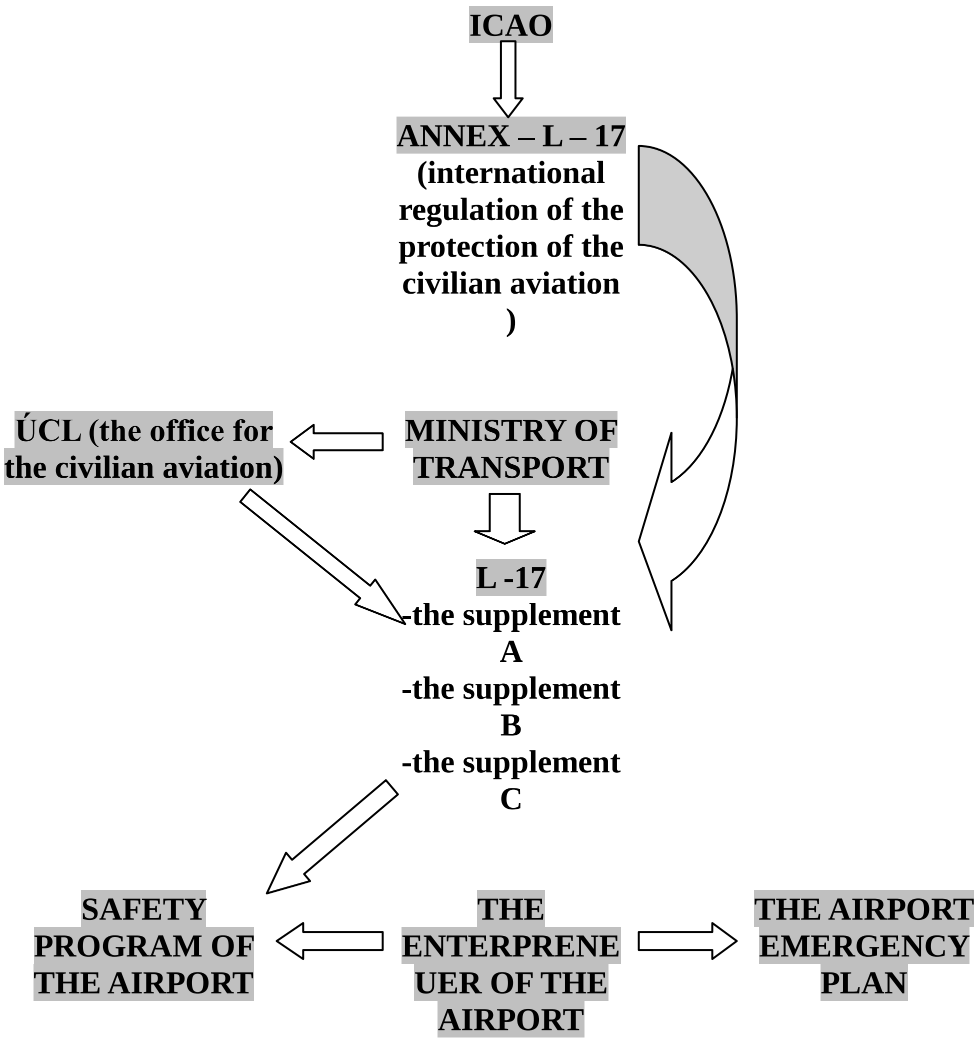
The international conventions and organizations:

- The convention about the international civilian aviation – 1944 (Chicago convention).
- Tokyo convention – 1963 (The problems of the illegal acts aboard a plane).
- Haag convention – 1970 (The repression and punishment of the hijacks).
- Montreal convention – 1971 (The repression and punishment of the illegal acts aboard a plane).
- The convention about marking of the plastic explosives – 1991 (This convention has not been described in the theoretical part, it is a regulation of the obligatory marking of the characteristically features of the explosives).
- The European conference for the civilian aviation – ECAC.
- The international association of the airline transporters – IATA.
- The European Union is here as the coordinator and via its authorities it orders obligatory norms for all its member states.
- The international organization for the civilian aviation - ICAO gives out so-called ANNEXES, which perform the individual norms and regulations for the civilian air transport.

The national regulations:

- The law about the civilian aviation 49/1997 Sb.
- The decree No. 410/2006 Sb. about the protection of the civilian aviation against the illegal acts.
- The airborne regulation ICAO – ANNEX – L-17 and the National safety program.
- The National safety program of the protection of the civilian aviation in the Czech Republic against the illegal acts.
- The National program of the safety exercise in the civilian aviation in the Czech Republic.

The picture 1 – The structural scheme of the legislation of the civilian aviation in the Czech Republic



Source: The safety program of the airport LKCS

D.11. THE DESCRIPTION OF RESULTS

For the calculation in the first phases the scaling was chosen, the aim of the measurement was to rank the individual respondents into the individual groups. In this case the division into the five elements of the scale according to the number of wrong answers was chosen. Further the chart with the calculation of the empirical distributions of frequencies was constructed (absolute, relative and cumulative), even here the features of Poisson distribution have displayed. By the calculation of the empirical parameters the important values for possible application of other way of the statistical inquiry were found out. The important values as standard deviation S_x give evidence how much the typical cases in the set of the studied numbers differ, further O_1 the raw moment, which is practically the arithmetical mean calculated for the individual sample statistical sets.

After the calculation of these values the comparison and testing of both sample statistical sets by the two sample t – test was processed. After the limiting the interval of the critical value of the t-test according to the statistical charts, filling in the necessary parameters in the equation and the calculation of the equation the result was found out. This result was compared and implemented into the critical region W. The result of the two sample t-test did not belong to the critical region, which meant the confirmation of the null hypotheses H_0 (both SSS lie on approximately same level of importance). The personnel of the airports LKCS and LKPR according to this testing have approximately same knowledge about the safety of the civilian aviation.

By the application of the Poisson distribution other calculation was executed. In the results the parameters needed to fill in the equation for the calculation of P_0 – $P_{4,}$ were calculated this equation can be substituted by the possibility of the usage of the computer server. Here there was substituted $\lambda = O_1$. After the calculation of the needed areas P_i the χ^2_{exp} was calculated. After the substitution and definition of the critical value χ^2_{krit} (statistical charts) it was found out that it is possible to replace the empirical distribution by the Poisson distribution. After the calculation of the experimental value of the test the last phase of the statistical testing followed which was the paired t-test which should compare the individual SSS in the area of Poisson distribution and the aim was to confirm or to disprove the null hypothesis. After the calculation of the necessary data (S_x, S_y) and after the substitution in the equation of the paired t – test the result was found out. After the setting of the critical interval of the t – test and the comparison of the result with the critical interval the null hypotheses H_0 was confirmed also in the region of the Poisson distribution. The hypothesis H1 can be unequivocally confirmed: The employees of the international airports LKPR and LKCS have approximately same knowledge in the field of safety of the civilian aviation.

The second part of the results – the structural analyses was based on the comparison of the legislative structures. The structural analyses of the legislative system guarantee the safety of the civilian aviation and the structural analyses of the legislative system at the international airport. The structure of the legislation was subsequently marked on the given scheme.

D.12. SUMMARY OF PART D

The submitted quantitative research report reflects the four scientific research degrees "reporting-exploration-explanation-prediction". It is only research report it means the total procedure of quantitative research could not be described. Some degrees were accentuated, some degree had to be suppressed due to the prescribed character of research report.

The base of the effective safety system in the field of civilian aircraft is consisting in motivated and continuously educated employees. This is valid not only for the employees of airports, airline companies and providers of other air services, but also for the policemen who are in charge to service at the international airports and other workers of the Police of the Czech Republic and Czech Ministry of the Interior who are engaged in this field.

The meaning of presented quantitative research **"Assessment of Safeguards to Protect Civil Aviation against Acts of Unlawful Interference at International Airports (especially focused on Ceske Budejovice Airport)"** is so obvious in the current volatile world (see also Appendix of submitted quantitative research) that a demonstration of its structure by research degrees deserves due attention.

This demonstration may be realized by means of the classification of individual paragraphs of research report according to scientific research degrees "**reporting-exploration-explanation-prediction**".

The first degree "Reporting" was only indicated through paragraph D.2. "Introduction". The simultaneous state of problem solving is by means of these paragraphs shown.

Considerable attention was devoted to the second degree "Exploration". It is possible to prove the mediation of the group of several following paragraphs of research report:

D.3. – Algorithm of statistical methods used

D.4. – Orientation of research report

D.5. – Operating hypotheses H1, H2

D.6. – Methodology

D.7. – Methodology – H1 statistical inquiry

D.8. – Methodology – H2 statistical inquiry

Considerable attention was also devoted to the third degree “Explanation”. It is possible again to prove the mediation of the following two chapters:

D.9. Results – H1 verification

D.10. Results – H2 verification

The fourth degree “Prediction” is emanating from paragraph D.11. “The description of results”. The necessity to investigate the effective safety system in the field of civilian aircraft can be demonstrated by the events occurring in the present (see also Appendix of submitted quantitative research).

Presented qualitative research is also issuing from the following sources: Anderlova (2001), ANNEX ICAO L-14 (2009), ANNEX ICAO L-17 (2011), Koverdynsky (2007), Kulcak, Kerner, Sykora (2003), Zaskodny, Havrankova, Havranek, Vurm (2011).

Key words

Quantitative research, Quantitative research degrees, Acts of Unlawful Interference of International Civil Aviation, Public International Airports, Non-public International Airports, Statistical Survey

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D.14. APPENDIX (the necessity of aviation safeguards research)

(quoted through en.wikipedia.org/wiki/Airport_security)

United States Airport Incidents

- On February 27, 2006, at the Will Rogers World Airport in Oklahoma City, in an airliner cargo area (accessible only to authorized personnel), threatening graffiti was found.

- On March 6, 2006 at John F. Kennedy International Airport in New York, an elderly man drove his car onto the runway through two security gates. He made it to an active runway where an Air France aircraft was preparing to land. The man drove around for approximately 23 minutes before being stopped. On the same day a man made it on to the runway by running through a secure gate while it was being opened at Midway International Airport in Chicago. The man made it through one of the three perimeter entrances that did not have a camera, resulting in four different runways being closed down. This incident led to 222 aviation security officers being retrained and a redesign of all perimeter gates.

- On March 11, 2006, after four years of continuous security breaches and staffing problems news reports indicated that federal officials removed the head of security at Newark Liberty International Airport.

- On November 1, 2013, a gunman named Paul Anthony Ciancia, age 23, shot and killed TSA agent Gerardo I. Hernandez at the Terminal 3 security checkpoint in Los Angeles International Airport. Ciancia then shot 2 other civilians as he passed security. He made it to the food court in the back of the terminal where federal agents tracked him down and shot him. He was then transported to a trauma hospital where he was treated for his injuries and will be released by the end of November 2 to law enforcement, most likely the FBI. The incident called for a complete shut down in the passenger drop off/departure roadways on the upper level of the airport and caused hundred of flight cancellations across the nation, although international flights had no cancellations. Terminals one, two, and three were shut down and so were the nearby runways, 24L and 24R. Flights that were already in the air and were preparing to land at these runways were either directed to land at the south runways or to divert to Ontario International Airport or Long Beach Airport.

E. STATISTICAL ANALYSIS OF EURO BUND FUTURES (GERMAN CFD) AND TWITTER INC. (NYSE)

E.1. QUANTITATIVE RESEARCHES AND THEIR DEGREES

This chapter consists of three quantitative researches – Normality Testing, Application of Black-Scholes Model and Binomial Model on the basis of the specific market. Normality Testing is focused on testing the zero hypothesis and the other researches are focused on hypotheses connected with option pricing.

In the framework of an individual quantitative research it is possible to identify four basic scientific research degrees (Zaskodny, Zaskodny, 2014):

- **Reporting** (input theory, input data),

- **Exploration** (the formulation of operating hypotheses with explanative functions and containing explorative variables “EV”),

- **Explanation** (the measurement of explorative variables "EV", the verification of hypotheses, the application of explanative functions of hypotheses),

- **Prediction** (the possible recommendations, the improvement of input theory).

Apart from relevant hypotheses it is possible to define the explorative variables for individual quantitative researches. The abbreviation "EV" will be used for explorative variable.

The first quantitative research is Normality Testing, where two variables are used to: EV-1 closing prices, EV-2 type of theoretical distribution N (μ; σ). The goal of this part is to validate the hypothesis H_0 that it is possible to replace the empirical distribution function of this financial derivative by the theoretical normal distribution function. Data for this part were chosen from German CFD exchange namely Euro Bund Futures. It was used χ^2 test and following formulas:

$$\chi^2_{exp} = \sum_{i=1}^{6} \frac{(n_i - np_i)^2}{np_i}; \ \chi^2_{th\,k-r-1}(\alpha = 0{,}05)$$

In this case $\chi^2_{exp} < \chi^2_{th}$ and it is showing that the empirical distribution can be substituted by the theoretical normal distribution and we can accept the null hypothesis (at the significant level 5%). The empirical graph can be replaced by Gauss curve. The supply corresponds with the demand of Euro Bund futures and there is no withholding information.

The second quantitative research is about application of the Black-Scholes Model on the basis of specific market (the NYSE in this case). The Black-Scholes model is used to find the optimal option price in the market. Needed variables are follows: S (Spot price), X (Exercise price), σ (Volatility), τ (Time to maturity) and r (free-risk annual rate) and type of theoretical distribution N (0; 1).The main goal of this thesis is to find the price of call and put option on share without dividend with an application of Black-Scholes The underlying asset of this options is share of Twitter inc, traded on the NYSE. The hypothesis H_1: The price of option according the Black-Scholes model is identifiable within that market. Four of these variables were found in the market data and the 5-Year Treasury Note of USA is considered for an free-risk annual rate. To calculate the price of call and put option were used the following formulas:

$$< C >= S \cdot N(d_1) - X \cdot e^{-r \cdot \tau} \cdot N(d_2), < P >= X \cdot e^{-r \cdot \tau} \cdot N(-d_2) - S \cdot N(-d_1)$$

$$< C >= \mathbf{1{,}8314} \, ; < P > = \mathbf{1{,}5996}$$

According the parity model was checked the resulting values:

$$< C > + PV(X) =< P > + S.$$

After adding parameters: 41,8314 ≅ 41,7896

If we compare the resulting values to values in the market we discover that the prices are approximately equal. Call option would cost 1,8314 according the test and price on the market is 1,85 - 1,87 (bid – ask). Put option cost on the market 1,6 – 1,62 (bid – ask). The prices on the market also confirmed our calculation.

The third quantitative research of this chapter is focused on application of the Binomial model on the basis of specific market (the same as in the second part). The Binomial model is used to find the optimal option price in the market, however the Black-Scholes model is considered to be a more accurate calculation. The main goal of this thesis is to find the price of call option on share without dividend with an application of Binomial model and create the Binomial tree. The underlying asset of these options is share of Twitter inc, traded on the NYSE. The hypothesis H_2: The price of option according the Binomial model is identifiable within that market.

In this model are explorative variables EV: S (Spot price), X (Exercise price), u (up/growth rate), d (down rate / rate of decrease) and q (free-risk interest rate for one period) and the type of theoretical distribution Bi (n; p). Only spot and exercise price have been indentified on the market data and it was necessary to calculate the other three variables.

$$u = e^{\sigma\sqrt{\Delta t}}\,;\, d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}\,;\, q = e^{r\cdot\Delta t}$$

For the calculation the call option was used the following formula:

$$< C > = \frac{1}{q^n} \cdot \sum_{j=0}^{n} \binom{n}{j} \cdot p^j \cdot (1-p)^{n-j} \cdot \max(0; S_j - X)$$

$$C_0^0 = \frac{1}{1{,}00021} \cdot (0{,}490199 \cdot C_1^1 + 0{,}509801 \cdot C_0^1) = 1{,}916813$$

In this part we do not calculate the price of put option and it was created the Binomial tree only for prices of call option. After finding results of these calculations, we can say that the price of the underlying asset should be 1,916813 based on Binomial model. The real price is closer to results from Black-Scholes model and for this reason it is recommended to prefer calculation of option prices by Black-Scholes model.

Three presented small quantitative researches have demonstrated the usefulness of four scientific research degrees "**reporting**", "**exploration**", "**explanation**", "**prediction**" application on the description of quantitative research structure.

E.2. NORMALITY TEST OF EURO BUND FUTURES

E.2.1. Introduction

For this quantitative research Euro Bund Futures on German CFD exchange were chosen. Euro Bund Futures are traded on German CFD exchange and the EUREX exchange. The abbreviation CFD means Contract For Difference – difference between the price of the underlying instrument on the day of opening and closing position – without physical delivery of the underlying instrument. This is one type of financial derivatives and this form of investment is possible only since 2006 in German. Before the arrival of online trading, most people living in Germany who wanted to trade CFDs chose to open accounts with English CFD dealers. Nowadays in German the number of „trading people“ with this instrument exceeded 70 000 (number from March 2011).[1]

χ^2 - test is used to determine whether the hypothesis is valid: empirical distribution function of this financial derivative can be replaced by the theoretical normal distribution function.

E.2.2. Empirical statistics

This research consists of an statistic analysis of the closing price development of this futures between 29th July 2014 and 21stOctober 2014 – the last 60 trading days. The statistical set has 60 measurements based on this data from exchange (the data are in Appendix 1).

Formulation of statistical analysis is based on the definition of these terms:

- Statistical set – 60 trading days
- Statistical symbol (variable) – closing price of futures
- Statistical unit – one trading day

E.2.3. Formulation of statistical measurement

Firstly, we have to know the range between the lowest and the highest value from the statistic set – width of interval and then we count the number of scales, which this set will have.

[1] CFDs in Germany. Contacts for Difference and CFDs Trading [online]. c2010-2011 [Retrieved 2014-10-22]. Http://www.contracts-for-difference.com/Germany-CFDs.html

$$k = 1 + 3{,}3\, log_{10}\, n$$

k – number of scales, n – range of statistic set

$$k = 1 + 3{,}3\, log_{10}\, 60 = 6{,}868$$

Based on the figure one we can choose six or seven scales. For this measurement were selected the six scales.

The figure to calculate the variation range is the second important figure, which we need to calculate before the measurement.

variation range = max x – min x

In our statistical set the maximum is equal to 151,16 and the minimum 147,88. The resulting number is divided to number of scales and then we determine the range of these scales.

Variation range = 151,16 – 147,88 = 3,93 → : 6 = 0,655

Range of scales are follows:

1: 147,88 – 148,54mu,

2: 148,54 – 149,19 mu,

3: 149,19 – 149,85 mu,

4: 149,85 – 150,50 mu,

5: 150,50 – 151,16 mu,

6: 151,16 – 151,81 mu.

Results of 60 measurements are listed in chart 1 rounded to 4 decimals. The first column presents scale, the second absolute frequency (n_i), the third relative frequency ($\frac{n_i}{n}$), the fourth cumulative relative frequency ($\sum \frac{n_i}{n}$) and the fifth to the eighth column shows values of empirical parameters – position, variability, skewness and sharpness.

Chart 1: Result of 60 measurements

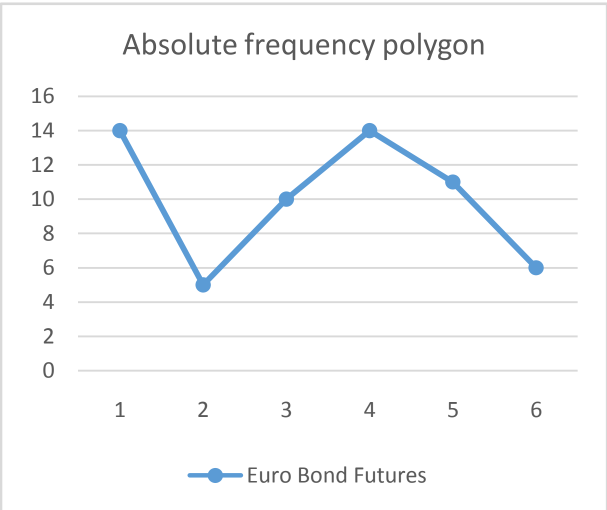
x_i	n_i	$\frac{n_i}{n}$	$\sum \frac{n_i}{n}$	$x_i n_i$	${x_i}^2 n_i$	${x_i}^3 n_i$	${x_i}^4 n_i$
1	14	0,2333	0,2333	14	14	14	14
2	5	0,0833	0,3167	10	20	40	80
3	10	0,1667	0,4833	30	90	270	810
4	14	0,2333	0,7167	56	224	896	3584
5	11	0,1833	0,9	55	275	1375	6875
6	6	0,1	1	36	216	1296	7776
	$\sum 60$	$\sum 1$		$\sum 201$	$\sum 839$	$\sum 3891$	$\sum 19139$

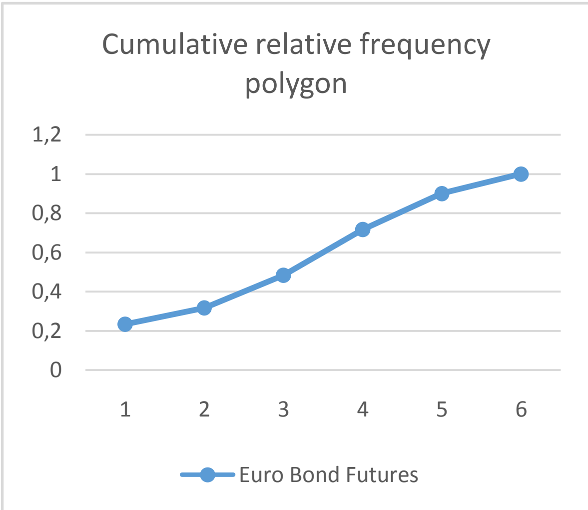
The number of elements in the range must be equal or higher then 5 at least of 80 % of scales. This condition is fulfilled.

E.2.4. Graphical expression of empirical distribution

The empirical frequency distribution can divide to two types. The first one refer the range of elements x_i corresponding absolute frequency n_i or relative frequency $\frac{n_i}{n}$. The second type refer the range of elements corresponding cumulative relative frequency $\sum \frac{n_i}{n}$.

The number of scales is entered on the x-axis and values of frequencies are on the y-axis (absolute or cumulative relative frequency).





Graph1: Graphs of empirical frequency distribution

E.2.5. Testing the hypothesis

The hypothesis H_0 was tested: empirical distribution function of this financial derivative can be replaced by the theoretical normal distribution function. Another variables are needed for the test of this hypothesis – position parameter O_1 (the weighted arithmetic average), parameter variability C_2(the empirical dispersion), standard deviation S_x (explanatory power of O_1) and variation coefficient V (how many percent S_x is involved in the O_1).

$$O_1 = \sum \frac{n_i}{n} x_i = \frac{201}{60} = 3{,}35$$

$$O_2 = \sum \frac{n_i}{n} {x_i}^2 = \frac{839}{60} = 13{,}9833$$

$$S_x = \sqrt{O_2 - O_1^2} = \sqrt{2{,}76} = 1{,}6616$$

$$V = \frac{S_x}{O_1} = \frac{1{,}6616}{3{,}35} = 0{,}496$$

E.2.6. χ^2 test

„χ^2 test may be defined as the sum of the squared difference of observed and expected frequencies divided by the expected frequency. It is a descriptive measure of the discrepancy values between observed frequency and expected frequency. The larger the discrepancies between these parameters, the larger the chi-square value obtained. If observed and expected frequencies show no discrepancies at all, the chi-square value is zero. Chi-square value is always a positive number.“[2]

First, we need to convert the descriptive statistics to mathematical statistics, which means that instead of the individual lines we obtained ranges. In this case we had 6 ranges and it corresponds to areas under the Gauss curve. Next, we convert the normal distribution to the theoretical normal distribution. We must convert limits of integrals.

Normal distribution has two parameters - the mean value μ, which characterizes the position of the distribution and variance σ (general moment of second order) characterizing the variability of the distribution.

[2]PALER-CALMORIN, Laurentina a Melchor A.CALMORIN. Statistics in Education and the Sciences. Philippines: Rex Book Store Inc., 1997. ISBN 971-23-2232-7.

Normal (Gauss) probability density function has the form:

$$f(x) = \rho\,(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

The theoretical normal probability density function has the mean value equals to zero and variance to one (μ = 0, σ = 1) and the curve is symmetrical about y-axis. Last step is introduction of new variable u replaced the $F(x)$ with distribution function $F(u)$and then the function is follows:

$$u = \frac{x-O_1}{S_x},$$

u – new variable, x – original limit of the integral, O_1 *– arithmetic average,* S_x *- standard deviation*

$$f(u) = \rho\,(u) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{u^2}{2}}$$

After the recalculation limits of the integrals we gain new limits, from which we calculate the content of the areas under the Gauss curve.

$$u_1 = \frac{1{,}5-3{,}35}{1{,}6616} = -1{,}11 \;\rightarrow\; p_1 = \int_{-\infty}^{-1{,}11} \rho_u\,du = 1 - F(1{,}11) = 0{,}1335$$

$$u_2 = \frac{2{,}5-3{,}35}{1{,}6616} = -0{,}51 \;\rightarrow\; p_2 = \int_{-1{,}11}^{-0{,}51} \rho_u\,du = [F(-0{,}51)] - F\,(-1{,}11) = 0{,}17153$$

$$u_3 = \frac{3{,}5-3{,}35}{1{,}6616} = 0{,}09 \;\rightarrow\; p_3 = \int_{-0{,}51}^{0{,}09} \rho_u\,du = F(0{,}09) - F(-0{,}51) = 0{,}23083$$

$$u_4 = \frac{4{,}5-3{,}35}{1{,}6616} = 0{,}69 \;\rightarrow\; p_4 = \int_{0{,}09}^{0{,}69} \rho_u\,du = F(0{,}69) - F(0{,}09) = 0{,}21904$$

$$u_5 = \frac{5{,}5-3{,}35}{1{,}6616} = 1{,}29 \;\rightarrow\; p_5 = \int_{0{,}69}^{1{,}29} \rho_u\,du = F(1{,}29 - F(0{,}69) = 0{,}14657$$

$$u_6 = \frac{\infty-3{,}35}{1{,}6616} = \infty \;\rightarrow\; p_6 = \int_{1{,}29}^{\infty} \rho_u\,du = 1 - F(1{,}29) = 0{,}09853$$

This new values must be in good range of the original values of the given interval, and this fact we must verify using χ^2-test. It consists of two parts – theoretical and experimental. If the experimental component has lower value than theoretical, then applies: empirical distribution can be substituted for by the theoretical normal distribution – Gauss curve and we can accept the null hypothesis H_0. In the opposite case experimental component has higher value than the theoretical and the hypothesis we cannot accept. The empirical distribution cannot be substituted for by the theoretical normal distribution.

$$\chi^2_{exp} = \sum_{i=1}^{6} \frac{(n_i - np_i)^2}{np_i}$$

$$\chi^2_{exp} = \frac{(14 - 60 \cdot 0{,}1335)^2}{60 \cdot 0{,}1335} + \frac{(5 - 60 \cdot 0{,}17153)^2}{60 \cdot 0{,}17153} + \frac{(10 - 60 \cdot 0{,}23083)^2}{60 \cdot 0{,}23083} + \frac{(14 - 60 \cdot 0{,}21904)^2}{60 \cdot 0{,}21904} + \frac{(11 - 60 \cdot 0{,}14657)^2}{60 \cdot 0{,}14657} + \frac{(6 - 60 \cdot 0{,}09853)^2}{60 \cdot 0{,}09853} = \mathbf{1{,}40909}$$

Experimental component shows the difference between lines and areas. We use significance level 5% (α = 0,05) for the theoretical component, because this level is sufficient in economy.$\chi^2_{th\,k-r-1}(\alpha = 0{,}05)$

k – number of scales, r – number of theoretical parameters (2 in our case)

$$\chi^2_{th\,k-r-1}(\alpha = 0{,}05) = \chi^2_{th\,6-2-1}(\alpha = 0{,}05) = \chi^2_{th\,3}(\alpha = 0{,}05) = \mathbf{7{,}81}$$

Now we compare the obtained values of experimental and theoretical component and it shows that experimental value is lower than the theoretical value.

Conclusion for normality test:

$\chi^2_{exp} < \chi^2_{th} \rightarrow H_0$: The empirical distribution can be substituted by the theoretical normal distribution and we can accept the null hypothesis at the significant level 5%. The empirical graph can be replaced by Gauss curve. The supply corresponds with the demand of Euro Bund futures and there is no withholding information.

E.3. APPLICATION OF BLACK-SCHOLES EQUATION AND BINOMIAL MODEL FOR OPTIONS PRICING

E.3.1. Black-Scholes Model

The main goal of this thesis is to find the price of call and put option on share without dividend with an application of Black-Scholes model and a binomial model. The underlying asset of these options is share of Twitter inc, traded on the NYSE.

New York Stock Exchange is Intercontinental Exchange's diverse markets. They are traded there futures, options on interest rates, commodities, indexes and FX as well as the equity options. This stock exchange has 5 clearing houses and it connects 5 exchanges. New York brokers establish the New York Stock & Exchange Board (NYS&EB) in 1817.[3]

The spot price of the underlying assets were detected to date 25th of November 2014 in the amount of 40,19 cu. The strike price of the underlying asset was identified in the amount of 40 cu. The time to option expiration is 24 days. The 5-Year Treasury Note of USA is considered for an free-risk annual rate 1,59 % in this paper[4].

The volatility of this underlying asset in the value of 41,7942 %[5].

The Black-Scholes model calculates the price of call and put option. The figures for this model are as follows:

$$< C >= S \cdot N(d_1) - X \cdot e^{-r \cdot \tau} \cdot N(d_2),$$

<C> - the price of call option, S – spot price, X – strike/exercise price, τ – time to maturity, N (d_1) and N(d_2) - values of standardized normal distribution, r – free-risk annual rate

$$< P >= X \cdot e^{-r \cdot \tau} \cdot N(-d_2) - S \cdot N(-d_1)$$

<P> - the price of put option.

E.3.2. Terminology

The basic parameters and factors (explorative variables EV) in the market are possible for analysis without specific figures. The lower is the strike price of call option, the higher is

[3]Intercontinental Exchange [online]. c2014 [Retrieved 2014-12-02]. Https://www.intercontinentalexchange.com/about
[4]United States Government Bonds. Bloomberg.com [online]. c2014 [Retrieved 2014-11-25].Http://www.bloomberg.com/markets/rates-bonds/government-bonds/us/
[5]Virtual Account Overview. CBOE.com [online]. 2014 [Retrieved 2014-11-25]. Https://www.cboe.com/VirtualTrading/VirtualTradeFramed.aspx?section=SEC_TRADING_TOOLS

the value of this option. The higher is the spot price of underlying asset, the higher is the value of call option. This principle is opposite for put option.

The spot price: The price at which a physical commodity for immediate delivery is selling at a given time and place. Also it is called cash price.

The strike price (or exercise price): The price, specified in the options contract, at which the underlying futures contract, security, or commodity will move from seller to buyer.

The volatility: A statistical measurement of the rate of price change of a futures contract, security, or other instrument underlying an option. Several types of volatility exist, e.g. implied volatility, historical volatility.[6]

The time to maturity is the remaining life of a debt instrument. In bonds, term to maturity is the time between when the bond is issued and when it matures (its maturity date), referred in years.

The risk-free interest rate is the theoretical rate of return of an investment with no risk of financial loss. One interpretation is that the risk-free rate represents the interest that an investor would expect from an absolutely risk-free investment over a given period of time.[7]

Chart 2*: Influences of parameters to value of call and put options*

Parameter	**Call option**	**Put option**
Spot price S	+	-
Strike price X	-	+
Time to maturity τ	+ american	+ american
Volatility σ	+	+
Free-risk annual rate r	+	-

E.3.3. Calculation

The values of above variables are:

S = 40,19

X = 40

[6]KERR, Kevin. A maniac commodity trader's guide to making a fortune: a not-so crazy roadmap to riches. Hoboken, N.J.: John Wiley, c2007, vi, 233 p. ISBN 04-717-7190-2.
[7]Investopedia.com [online]. c2014 [Retrieved 2014-12-02]. Http://www.investopedia.com/

τ = 24 days = $\frac{24}{365} = 0{,}0658$ year

r = 1,59 % = 0,0159

To calculate the price of call and put option we need another figures – for d_1 and d_2 and determine the volatility (= 0,4179).

$$d_1 = \frac{\ln\frac{S}{X} + (r + \frac{\sigma^2}{2}) \cdot \tau}{\sigma \cdot \sqrt{\tau}}, d_2 = d_1 - \sigma \cdot \sqrt{\tau}$$

σ – volatility (in this case implied volatility)

According the above figure, we calculated the value of these parameters:

$$d_1 = \frac{\ln\frac{40{,}19}{40} + (0{,}0159 + \frac{0{,}4179^2}{2}) \cdot 0{,}0658}{0{,}4179 \cdot \sqrt{0{,}0658}} = 0{,}107564$$

$$d_2 = 0{,}107564 - 0{,}4179 \cdot \sqrt{0{,}0658} = 0{,}000367$$

Now we have to find out the values of theoretical normal distribution function from the statistical tables. It is necessary to round the values: $d_1 = 0{,}11$ and $d_2 = 0{,}00$.

$$N(d_1) = 0{,}54380, N(d_2) = 0{,}5$$

$$N(-d_1) = 0{,}4562;\ N(-d_2) = 0{,}5$$

Finally, can substitute the values into the main formula and gain the first goal of this chapter: determined the price of call and put option.

$$< C >= 40{,}19 \cdot 0{,}543\,80 - 40 \cdot e^{-0{,}0159 \cdot 0{,}0658} \cdot 0{,}5 = 1{,}876$$

$< P >= 40 \cdot e^{-0{,}0159 \cdot 0{,}0658} \cdot 0{,}5 - 40{,}19 \cdot 0{,}4562 = 1{,}6444$

These values were calculated based on statistical tables. For the control of these values, we put the parameters into the Black-Scholes model calculator.[8] Then the real values are more precise results:

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[8]Black Scholes Calculator. Money-zine.com [online]. c2009-2011 [Retrieved 2014-11-25]. Http://www.money-zine.com/calculators/investment-calculators/black-scholes-calculator

$$< C >= \mathbf{1,8314}\ and < P > = \mathbf{1,5996}$$

The other type of controlling is this figure:

$$< C > + PV(X) =< P > + S$$

After adding parameters:

$$1{,}8314 + 40{,}00 = 1{,}5996 + 40{,}19$$

$$41{,}8314 \cong 41{,}7896$$

E.3.4. Conclusion for Black-Scholes model

When the underlying asset is the share of Twitter Inc., traded on the New York Stock Exchange, the price of call option is 1,8314 and price of put option is 1,5996 based on Black-Scholes model.

If we compare the resulting values to values in the market we discover that the prices are approximately equal. Call option would cost 1,8314 according the test and price on the market is 1,85 – 1,87 (bid – ask). Put option cost on the market 1,6 – 1,62 (bid – ask). The prices on the market also confirmed our calculation.

E.3.5. Binomial Tree for TWTR option pricing

We must determine the index of growth, rate of the decrease and recalculate the free-risk annual rate to free-risk interest rate for one period to construct the binomial tree. For this parameters are follows figures:

$q = e^{r \cdot \Delta t}$ – free-risk interest rate for one period

$u = e^{\sigma\sqrt{\Delta t}}$ – up rate / growth rate

$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$ down rate / rate of decrease

$$\Delta t = \frac{\tau}{n}$$

$$q = e^{0{,}0159 \cdot \frac{0{,}0658}{5}} = 1{,}00021; \quad u = e^{0{,}4179 \cdot \sqrt{\frac{0{,}0658}{5}}} = 1{,}04911 \;\; d = \frac{1}{1{,}049108} = 0{,}95319$$

The same goal as in the previous research is in this part – to determine the price of put and call option for the underlying asset share of Twitter Inc. In this part we will calculate and construct binomial tree only for call option. The basic formulas are:

$$< C > = \frac{1}{q^n} \cdot \sum_{j=0}^{n} \binom{n}{j} \cdot p^j \cdot (1-p)^{n-j} \cdot \max(0; S_j - X)$$

$$< P > = \frac{1}{q^n} \cdot \sum_{j=0}^{n} \binom{n}{j} \cdot p^j \cdot (1-p)^{n-j} \cdot \max(0; X - S_j)$$

n – number of period (=5)

p –probability of an up move in the underlying asset

1-p – probability of a down move in the underlying asset

j –number of period where the price of underlying asset is growing

X – strike price, S_j *– price of the underlying asset on j-growth*

q – free-risk interest rate for one period

Formula to construct a probability of up/down move in the underlying asset:

$$p = \frac{q-d}{u-d}$$

$$p = \frac{1{,}00021 - 0{,}95319}{1{,}04911 - 0{,}95319} = 0{,}490199$$

First tree shows the price of underlying asset on j-growth. We use this figure for the calculation of these prices:

$$S_j^k = S \cdot u^j \cdot d^{k-j}$$

k- ranking of period

Table 1: Binomial tree of Spot prices

S(5,5)		S(5,4)		S(5,3)		S(5,2)		S(5,1)		S(5,0)
51,07624		46,40647		42,16365		38,30874		34,80627		31,62402
	S(4,4)		S(4,3)		S(4,2)		S(4,1)		S(4,0)	
	48,6854		44,23422		40,19		36,51553		33,17701	
		S(3,3)		S(3,2)		S(3,1)		S(3,0)		
		46,40647		42,16365		38,30874		34,80627		
			S(2,2)		S(2,1)		S(2,0)			
			44,23422		40,19		36,51553			
				S(1,1)		S(1,0)				
				42,16365		38,30874				
					S(0,0)					
					40,19					

$$S_1^1 = 40{,}19 \cdot 1{,}04911^1 \cdot 0{,}95319^0 = 42{,}16365$$

$$S_0^1 = 40{,}19 \cdot 1{,}04911^0 \cdot 0{,}95319^1 = 38{,}30874$$

$$S_2^2 = 40{,}19 \cdot 1{,}04911^2 \cdot 0{,}95319^0 = 44{,}23422$$

$$S_1^2 = 40{,}19 \cdot 1{,}04911^1 \cdot 0{,}95319^1 = 40{,}19$$

$$S_0^2 = 40{,}19 \cdot 1{,}04911^0 \cdot 0{,}95319^2 = 36{,}51553$$

$$S_3^3 = 40{,}19 \cdot 1{,}04911^3 \cdot 0{,}95319^0 = 46{,}40647$$

$$S_2^3 = 40{,}19 \cdot 1{,}04911^2 \cdot 0{,}95319^1 = 42{,}16365$$

$$S_1^3 = 40{,}19 \cdot 1{,}04911^1 \cdot 0{,}95319^2 = 40{,}19$$

$$S_0^3 = 40{,}19 \cdot 1{,}04911^0 \cdot 0{,}95319^3 = 34{,}80627$$

$$S_4^4 = 40{,}19 \cdot 1{,}04911^4 \cdot 0{,}95319^0 = 48{,}6854$$

$$S_3^4 = 40{,}19 \cdot 1{,}04911^3 \cdot 0{,}95319^1 = 44{,}23422$$

$$S_2^4 = 40{,}19 \cdot 1{,}04911^2 \cdot 0{,}95319^2 = 40{,}19$$

$$S_1^4 = 40{,}19 \cdot 1{,}04911^1 \cdot 0{,}95319^3 = 36{,}51553$$

$$S_0^4 = 40{,}19 \cdot 1{,}04911^0 \cdot 0{,}95319^4 = 33{,}17701$$

$$S_5^5 = 40{,}19 \cdot 1{,}04911^5 \cdot 0{,}95319^0 = 51{,}07624$$

$$S_4^5 = 40{,}19 \cdot 1{,}04911^4 \cdot 0{,}95319^1 = 46{,}40647$$

$S_3^5 = 40{,}19 \cdot 1{,}04911^3 \cdot 0{,}95319^2 = 42{,}16365$

$S_2^5 = 40{,}19 \cdot 1{,}04911^2 \cdot 0{,}95319^3 = 38{,}30874$

$S_1^5 = 40{,}19 \cdot 1{,}04911^1 \cdot 0{,}95319^4 = 34{,}80627$

$S_0^5 = 40{,}19 \cdot 1{,}04911^0 \cdot 0{,}95319^5 = 31{,}62402$

Now we know prices of call option, but no the probability of each possibilities.

Table 2: Binomial tree of Spot prices with percentages

S(5,5)		S(5,4)		S(5,3)		S(5,2)		S(5,1)		S(5,0)
51,07624		46,40647		42,16365		38,30874		34,80627		31,62402
2,83%		14,72%		30,61%		31,84%		16,56%		3,44%
	S(4,4)		S(4,3)		S(4,2)		S(4,1)		S(4,0)	
	48,6854		44,23422		40,19		36,51553		33,17701	
	5,77%		24,02%		37,47%		25,98%		6,75%	
		S(3,3)		S(3,2)		S(3,1)		S(3,0)		
		46,40647		42,16365		38,30874		34,80627		
		11,78%		36,75%		38,22%		13,25%		
			S(2,2)		S(2,1)		S(2,0)			
			44,23422		40,19		36,51553			
			24,03%		49,98%		25,99%			
				S(1,1)		S(1,0)				
				42,16365		38,30874				
				49,02%		50,98%				
					S(0,0)					
					40,19					

$$p_j^k = \binom{k}{j} \cdot p^j \cdot (1-p)^{k-j}$$

$p_5^5 = \binom{5}{5} \cdot 0{,}490199^5 \cdot 0{,}509801^0 = 2{,}83\ \%$

$p_4^5 = \binom{5}{4} \cdot 0{,}490199^4 \cdot 0{,}509801^1 = 14{,}72\ \%$

$p_3^5 = \binom{5}{3} \cdot 0{,}490199^3 \cdot 0{,}509801^2 = 30{,}61\ \%$

$p_2^5 = \binom{5}{2} \cdot 0{,}490199^2 \cdot 0{,}509801^3 = 31{,}84\ \%$

$p_1^5 = \binom{5}{1} \cdot 0{,}490199^1 \cdot 0{,}509801^4 = 16{,}56\ \%$

$p_0^5 = \binom{5}{0} \cdot 0{,}490199^0 \cdot 0{,}509801^5 = 3{,}44\ \%$

$$p_4^4 = \binom{4}{4} \cdot 0{,}490199^4 \cdot 0{,}509801^0 = 5{,}77\ \%$$

$$p_3^4 = \binom{4}{3} \cdot 0{,}490199^3 \cdot 0{,}509801^1 = 24{,}02\ \%$$

$$p_2^4 = \binom{4}{2} \cdot 0{,}490199^2 \cdot 0{,}509801^2 = 37{,}47\ \%$$

$$p_1^4 = \binom{4}{1} \cdot 0{,}490199^1 \cdot 0{,}509801^3 = 25{,}98\ \%$$

$$p_0^4 = \binom{4}{0} \cdot 0{,}490199^0 \cdot 0{,}509801^4 = 6{,}75\ \%$$

$$p_3^3 = \binom{3}{3} \cdot 0{,}490199^3 \cdot 0{,}509801^0 = 11{,}78\ \%$$

$$p_2^3 = \binom{3}{2} \cdot 0{,}490199^2 \cdot 0{,}509801^1 = 36{,}75\ \%$$

$$p_1^3 = \binom{3}{1} \cdot 0{,}490199^1 \cdot 0{,}509801^2 = 38{,}22\ \%$$

$$p_0^3 = \binom{3}{0} \cdot 0{,}490199^0 \cdot 0{,}509801^3 = 13{,}25\ \%$$

$$p_2^2 = \binom{2}{2} \cdot 0{,}490199^2 \cdot 0{,}509801^0 = 24{,}03\ \%$$

$$p_1^2 = \binom{2}{1} \cdot 0{,}490199^1 \cdot 0{,}509801^1 = 49{,}98\ \%$$

$$p_0^2 = \binom{2}{0} \cdot 0{,}490199^0 \cdot 0{,}509801^2 = 25{,}99\%$$

$$p_1^1 = \binom{1}{1} \cdot 0{,}490199^1 \cdot 0{,}509801^0 = 49{,}02\ \%$$

$$p_0^1 = \binom{1}{0} \cdot 0{,}490199^0 \cdot 0{,}509801^1 = 50{,}98\ \%$$

The second step to complete this Binomial tree is to construct tree with prices of call option in *k*-period and *j*-number of up moves.

$$C_j^k = \frac{1}{q} \cdot \left(p \cdot C_{j+1}^{k+1} + (1-p) \cdot C_j^{k+1}\right)$$

C – price of call option in k-period and j-number of up moves

X = 40

$$IVC_j^k = \max(0;\ S_j - X)$$

IVC_j^k shows the intrinsic value of the call option. There are three possibilities:

$IVC_j^k > 0$ →„In the money“ and call option brings profit

$IVC_j^k = O_{1;}\, S_j = X$ → „At the money“ and the option has same spot and strike price

$IVC_j^k = 0$ → „Out of money“

Table 3: Intrinsic value of the call option

C(5,5)		C(5,4)		C(5,3)		C(5,2)		C(5,1)		C(5,0)
11,07624		6,406474		2,163649		0		0		0
	C(4,4)		C(4,3)		C(4,2)		C(4,1)		C(4,0)	
	8,69377		4,24259		1,060397		0		0	
		C(3,3)		C(3,2)		C(3,1)		C(3,0)		
		6,42321		2,619756		0,519697		0		
			C(2,2)		C(2,1)		C(2,0)			
			4,483267		1,54882		0,254701			
				C(1,1)		C(1,0)				
				2,986658		0,888891				
					C(0,0)					
					1,916813					

$C_5^5;\ C_4^5;\ C_3^5;\ C_2^5;\ C_1^5;\ C_0^5: S_j - X\ or\ 0$

C_5^5 =51,07624 – 40 = 11,07624

C_4^5 =46,40647 – 40 = 6,406474

C_3^5 =42,16365 – 40 = 2,16365

$C_2^5 = C_1^5 = C_0^5 = 0$

$$C_4^4 = \frac{1}{1{,}00021} \cdot \left(0{,}490199 \cdot C_5^5 + 0{,}509801 \cdot C_4^5\right) = 8{,}69377$$

$$C_3^4 = \frac{1}{1{,}00021} \cdot (0{,}490199 \cdot C_4^5 + 0{,}509801 \cdot C_3^5) = 4{,}24259$$

$$C_2^4 = \frac{1}{1{,}00021} \cdot (0{,}490199 \cdot C_3^5 + 0{,}509801 \cdot C_2^5) = 1{,}060397$$

$$C_1^4 = \frac{1}{1{,}00021} \cdot (0{,}490199 \cdot C_2^5 + 0{,}509801 \cdot C_1^5) = 0$$

$$C_0^4 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_1^5 + 0,509801 \cdot C_0^5) = 0$$

$$C_3^3 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_4^4 + 0,509801 \cdot C_3^4) = 6,42321$$

$$C_2^3 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_3^4 + 0,509801 \cdot C_2^4) = 2,619756$$

$$C_1^3 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_2^4 + 0,509801 \cdot C_1^4) = 0,519697$$

$$C_0^3 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_1^4 + 0,509801 \cdot C_0^4) = 0$$

$$C_2^2 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_3^3 + 0,509801 \cdot C_2^3) = 4,483267$$

$$C_1^2 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_2^3 + 0,509801 \cdot C_1^3) = 1,54882$$

$$C_0^2 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_1^3 + 0,509801 \cdot C_0^3) = 0,254701$$

$$C_1^1 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_2^2 + 0,509801 \cdot C_1^2) = 2,986658$$

$$C_0^1 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_1^2 + 0,509801 \cdot C_0^2) = 0,888891$$

$$C_0^0 = \frac{1}{1,00021} \cdot (0,490199 \cdot C_1^1 + 0,509801 \cdot C_0^1) = 1,916813$$

It is not necessary to calculate *C*, when we know that it is negative value.

E.3.6. Conclusion for Binomial tree model

We have calculated the price of underlying asset on j-growth and probability of each possibilities, price of call option in *k*-period and *j*-number of up moves and intrinsic value of the call option. After finding results of these calculations, we can say that the price of the underlying asset should be 1,916813 based on binomial model.

E.4. SUMMARY OF PART E

Normality test (the first small quantitative research): $\chi^2_{exp} < \chi^2_{th} \rightarrow H_0$: The empirical distribution can be substituted by the theoretical normal distribution and we can accept the null hypothesis at the significant level 5%. The empirical graph can be replaced by Gauss curve. The supply corresponds with the demand of Euro Bund futures and there is no withholding information.

Black-Scholes model (the second small quantitative research – the hypothesis focused on BSM option pricing) : When the underlying asset is the share of Twitter Inc., traded on the New York Stock Exchange, the price of call option is 1,8314 and price of put option is 1,5996 based on Black-Scholes model. If we compare the resulting values to values in the market we discover that the prices are approximately equal. Call option would cost 1,8314 according the test and price on the market is 1,85 – 1,87 (bid – ask). Put option cost on the market 1,6 – 1,62 (bid – ask). The prices on the market also confirmed our calculation.

Binomial model the third small quantitative research – the hypothesis focused on BM option pricing): We have calculated the price of underlying asset on j-growth and probability of each possibilities, price of call option in k-period and j-number of up moves and intrinsic value of the call option. After finding results of these calculations, we can say that the price of the underlying asset should be 1,916813 based on binomial model.

The real price is closer to results from Black-Scholes model and for this reason it is recommended to prefer calculation of option prices by Black-Scholes model.

On the basis of investigation of three small quantitative researches the significance of scientific research degrees “**Reporting**”, “**Exploration**”, “**Explanation**”, “**Prediction**” has been demonstrated.

Presented quantitative researches are also issuing from sources Real Time Streaming Financial Futures (2014), Zaskodny, Pavlat, Budik (2007).

Keywords: Euro Bund Futures,Chi-square, distribution function, theoretical normal distribution, TWTR, Binomial tree, Black-Scholes model, scientific research degrees

JEL: C12, D49, G3

E.5. REFERENCES

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E.6. APPENDIX (data collection)

Date	Last
Jul 29, 2014	148,69
Jul 30, 2014	148,10
Jul 31, 2014	147,99
Aug 01, 2014	148,42
Aug 04, 2014	148,46
Aug 05, 2014	148,01
Aug 06, 2014	148,82
Aug 07, 2014	149,24
Aug 08, 2014	149,48
Aug 11, 2014	149,34
Aug 12, 2014	149,37
Aug 13, 2014	149,73
Aug 14, 2014	149,92
Aug 15, 2014	150,41
Aug 18, 2014	149,91
Aug 19, 2014	150,11
Aug 20, 2014	150,32
Aug 21, 2014	150,21
Aug 22, 2014	150,27
Aug 25, 2014	150,73
Aug 26, 2014	150,82
Aug 27, 2014	151,30
Aug 28, 2014	151,58
Aug 29, 2014	151,54
Sep 01, 2014	151,57
Sep 02, 2014	151,09
Sep 03, 2014	150,84
Sep 04, 2014	150,65
Sep 05, 2014	151,11
Sep 08, 2014	151,30

Date	Last
Sep 09, 2014	148,40
Sep 10, 2014	148,38
Sep 11, 2014	148,39
Sep 12, 2014	147,90
Sep 15, 2014	148,17
Sep 16, 2014	148,20
Sep 17, 2014	148,40
Sep 18, 2014	**147,88**
Sep 19, 2014	148,49
Sep 22, 2014	148,76
Sep 23, 2014	148,93
Sep 24, 2014	149,01
Sep 25, 2014	149,37
Sep 26, 2014	149,44
Sep 29, 2014	149,52
Sep 30, 2014	149,70
Oct 02, 2014	150,12
Oct 03, 2014	149,82
Oct 06, 2014	150,12
Oct 07, 2014	150,11
Oct 08, 2014	150,20
Oct 09, 2014	150,21
Oct 10, 2014	150,40
Oct 13, 2014	150,37
Oct 14, 2014	150,91
Oct 15, 2014	**151,81**
Oct 16, 2014	151,12
Oct 17, 2014	150,64
Oct 20, 2014	150,78
Oct 21, 2014	150,58

SUMMARY OF MONOGRAPH

Presented monograph "Application of Quantitative Research Degrees" shows the new continuities among elements of scientific research methodology. These new continuities are represented by means of the scientific research degrees.

Based on the described new continuities within scientific research methodology it is possible to deduce that quantitative research can be characterized by the degrees of

reporting-exploration-explanation-prediction,

while a typical sequence of qualitative research is of

reporting-exploration-interpretation-prediction.

Within Part A "The way of Degrees Use within Scientific Research" of monograph these scientific research degrees are connected by the algorithm of quantitative research as the needful arrangement the described scientific methods into the sequence of algorithmic steps. The algorithm of qualitative research can be found in Zaskodny, Zaskodna, 2014.

The typical sequence of qualitative research **"reporting-exploration-interpretation-prediction"** was illustrated by research of wave corpuscular duality through "de Broglie history" (see Part A of monograph), the typical sequence of quantitative research **"reporting-exploration-explanation-prediction"** was shown by research of linear operators in Hilbert space through "New quantum theory history" (see Part A of monograph).

Within Part B three small quantitative researches were presented. The normality test of selected financial derivative in the framework of Chicago Mercantile Exchange has formed the first small quantitative research. The right prices of call and put options have been calculated for the same financial derivative within the second small quantitative research (through Black-Scholes model) and the third small quantitative research (through binomial model).

The application of the quantitative researches degrees of **reporting-exploration-explanation-prediction** was demonstrated also through Part B in the framework of three research reports associated with above mentioned three small quantitative researches.

Within Part C the application of curricular process theory was submitted by means of the investigation of physics principles of magnetic resonance. The presented research report has demonstrated the structure of quantitative research which was again connected with the quantitative research degrees of **reporting-exploration-explanation-prediction**.

The progressive application of individual degrees has enabled to explain the physics principles of magnetic resonance through the linear operators defined within new quantum theory. In the course of mentioned explanation the explanatory functions of formulated operating hypotheses have been used.

Within Part D the application of population protection theory was submitted by means of the investigation of safeguards assessment for civil aviation at international airports**.** The presented research report has demonstrated the structure of quantitative research which was again associated with the quantitative research degrees of **reporting-exploration-explanation-prediction**.

Within Part D the first degree "Reporting" was only indicated through paragraph D.2. "Introduction". The simultaneous state of problem solving was by means of these paragraphs shown.

Considerable attention was devoted within Part D to the second degree "Exploration". It was proved the mediation of the group of several following paragraphs of research report:

D.3. – Algorithm of statistical methods used
D.4. – Orientation of research report
D.5. – Operating hypotheses H1, H2
D.6. – Methodology
D.7. – Methodology – H1 statistical inquiry
D.8. – Methodology – H2 statistical inquiry

Considerable attention was also devoted within Part D to the third degree "Explanation". It was proved the mediation of the explanatory functions of operating hypotheses H1, H2 and the mediation of the following two paragraphs:

D.9. Results – H1 verification
D.10. Results – H2 verification

Within Part D the fourth degree "Prediction" is emanating from paragraph D.11. "The description of results". The necessity to investigate the effective safety system in the field of civil aviation was demonstrated by the events occurring in the present (see Appendix of submitted quantitative research of safeguards assessment within civil aviation at international airports).

Within Part E three small quantitative researches were presented. The normality test of selected financial derivative in the framework of German CFD has formed the first small quantitative research. The right prices of call and put options have been calculated for the financial derivative Twitter Inc. (New York Stock Exchange) within the second small quantitative research (through Black-Scholes model) and the third small quantitative research (through binomial model).

The application of the quantitative researches degrees of **reporting-exploration-explanation-prediction** was demonstrated also through Part E in the framework of three research reports associated with above mentioned three small quantitative researches.

Key words – Part A of monograph
Degrees of scientific research, Qualitative and quantitative research, Hypothesis, Explanatory function, Interpretive function, Algorithm of quantitative research, Illustration of scientific research degrees

Key words – Part B of monograph
Scientific research degrees, Binomial options pricing model, Binomial tree, Black-Scholes model, Call option, Chi-square, Index E-mini S&P 500, Central moment, Distribution function, General moment, Greeks, Put option, Random variable, Standardized moment, Theoretical normal distribution.

Key words – Part C of monograph
Curricular process, Magnetic resonance, New quantum theory, Operators, Schrödinger equation, Physical bases for Radiographers, Conceptual curriculum, Intended curriculum, Projected curriculum, Implemented curriculum, Quantitative research, Degrees of quantitative research

Key words – Part D of monograph
Quantitative research, Quantitative research degrees, Acts of Unlawful Interference of International Civil Aviation, Public International Airports, Non-public International Airports, Statistical Survey

Key words – Part E of monograph
Euro Bund Futures, Chi-square, Distribution function, Theoretical normal distribution, TWTR, Binomial tree, Black-Scholes model, Scientific research degrees

SUBJECT INDEX

E

F

G

H

I

P

Q

R

S

NAME INDEX

AUTHORS, REVIEWERS

CVs OF AUTHORS

Assoc.Prof. RNDr.Premysl Zaskodny, CSc.

Assoc.Prof. RNDr.Premysl Zaskodny,CSc., graduated from the Mathematical-Physics Faculty of Charles University, CSc. in the physics education, and docent (assoc. professor) of physics education.

As a university teacher, he is affiliated to the University of South Bohemia in České Budějovice and to the University of Finance and Administration in Prague.

He is active in scientific work in cooperation with the International Institute of Informatics and Systemics in U.S.A., and the Curriculum Studies Research Group in Slovakia.

In his scientific work, aimed at science and statistics education, he deals with structuring and modeling physics and statistics knowledge and systems of knowledge and also data mining and curricular process.

In addition to support from his faculty and university, the projects granted to the author by the Avenira Foundation in Switzerland and the Curriculum Studies Research Group in Slovakia has brought a considerable contribution to the results achieved.

The conception of the last books

"Curricular Process of Physics" (2009),
"Fundaments of Statistics" (with co-authors, 2011),
"From Financial Derivatives to Option Hedging" (with co-author, 2012),
"The Principles of Probability and Statistics" (2013),
"Survey of Principles of Theoretical Physics" (with co-author, 2014)
"Methodology of Scientific Research" (with co-author, 2014)

and last monographs

"Educational and Didactic Communication 2008, 2009, 2010, 2011, 2012, 2013"

is based on the scientific work of the author.

Assoc.Prof. RNDr. Premysl Zaskodny, CSc. is active as general chair of international e-conferences OEDM-SERM 2011, OEDM-SERM 2012 and OEDM-SERM 2013/2014 (Optimization, Education and Data Mining in Science, Engineering and Risk Management).

Bc.Galina Ruban (bachelor´s degree)

Bc.Galina Ruban has finished the studies of banking and insurance at the Finance and Accounting Faculty of Economics University, Prague, Czech Republic. She has graduated from bachelor program of Banking and Insurance at the Economics University in 2014.

Her language abilities can be expressed by the following possibilities: Russian (native speaker), English (advanced), Czech (advanced), French (intermediate).

Currently she is studying within master program Finance and Financial Services at the Economic Faculty of Finance and Administration University, Prague, Czech Republic.

The specialization of her investigation is associated with the area of financial markets in United States of America.

Publications:

- Ruban,G., Bachelor Thesis (2014). *The comparison of bank fees in Czech Repubic and Russian Federation*
Prague, Czech Republic: Economics University (Informational System)

Bc.Lukas Fil (bachelor´s degree)

Bc.Lukas Fil has finished the study of ionizing radiation application at the Health and Social Studies Faculty of South Bohemia University, Ceske Budejovice, Czech Republic. He has graduated from bachelor program of Radiological Assistant at the Health and Social Studies Faculty of South Bohemia University in 2014.

Currently he works as diagnostic radiographer in Rudolf and Stefanie Hospital, Benesov, Czech Republic, Department of Radiodiagnostics.

Publications:

- Fil,L., Thesis (2014). *The Physics Principles of Magnetic Resonance for Radiographers*
  Ceske Budejovice, Czech Republic: South Bohemia University (Informational System)
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  Prague, Czech Republic: Curriculum
  ISBN 978-80-87894-01-9

Mgr.Jiri Urban (master´s degree)

Mgr.Jiri Urban has finished the study of inhabitants protection at the Health and Social Studies Faculty of South Bohemia University, Ceske Budejovice, Czech Republic. He has graduated from the bachelor program of Population Protection at the South Bohemia University in 2010.

Mgr. Jiri Urban has finished the follow-up study of inhabitants protection at the Health and Social Studies Faculty of South Bohemia University, Ceske Budejovice, Czech Republic. He has graduated from the master program of Civil Emergency Preparedness at the South Bohemia University in 2012.

His former work experiences can be expressed by the positions of the member of Fire Rescue in Prague, Czech Republic and of the member of Fire Rescue in Czech Krumlov, Czech Republic.

Currently he is employed as the first sergeant of Fire Brigade South Bohemia Region. He also works as the lecturer of diving courses organized by Fire Rescue and by Fire and Rescue Service at South Bohemia University.

Publications:

- Urban,J., Bachelor Thesis (2010). *Intervention of IZS to escape petroleum products to the dam Rimov*
Ceske Budejovice, Czech Republic: South Bohemia University (Informational System)
- Urban,J., Master Thesis (2012). *Assessment of security measures to protect civil aviation security at international airports in the Czech Republic*
Ceske Budejovice, Czech Republic: South Bohemia University (Informational System)
- Urban,J. (2010). *Training Fire Rescue Divers*
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- Urban,J. (2011). *The curriculum specialization course for divers, diving with Nitroxbreathing*
In: Fire Rescue, DG, No. MV-116584-1/PO-PVP-2011

Bc.Marketa Korandova (bachelor’s degree)

Bc.Marketa Korandova has finished the study of business management at the Economics Faculty of West Bohemia University, Pilsen, Czech Republic. She has graduated from the bachelor program of Business Management at the West Bohemia University in 2014.

She has also graduated from the intensive English language courses at the Spell Language School, Czech Republic and the Scanbrit School of English, United Kingdom. Her language skills can be described by the achieved levels: Czech (native speaker), French (A2), English (B2), Italian (A1).

Her former work experiences can be expressed by the positions of the personal assistant in the consulting company K2 Finance, Pilsen, Czech Republic and of the advisor for financial planning in ING, Pilsen, Czech Republic.

Currently she is studying within master program Finance and Financial Services at the Economic Faculty of Finance and Administration University, Prague, Czech Republic. In the framework of Erasmus program the summer semester at the Wroclaw University, Poland will be realized. The orientation of her exploration is connected with the area of financial markets.

CVs OF REVIEWERS

Assoc.Prof. PhDr.Helena Zaskodna,CSc.

Assoc.Prof. PhDr.Helena Zaskodna,CSc. has finished the studies of psychology at the Arts Faculty of Palacky University, since 1973 working as a university researcher and educationalist. She has graduated at the Arts Faculty of Palacky University in Czech republic and the Institute of Psychology in Katowice in Poland.

In the field of science she focuses on social psychology, psychopathology, clinical psychology, and selected issues of social pathology.

She worked at the Department of psychology of the Faculty of Arts the University of Ostrava. Currently she works at the Department of adult education and personnel management of the Arts Faculty of Charles University in Prague and the Department of psychology and special education of the Health and Social Studies Faculty of South Bohemia University in Ceske Budejovice.

As principal investigator or co-investigator she had participated in a series of scientific researches (FRVŠ, Foundation of J.Hus in Brno, Tempus, MŠMT ČR, Publisher Didaktis, GAČR, COST).

More than 90 studies published in national and international scientific and professional journals, she is the author of the scientific monograph, co-author of 6 monographs, author and co-author of 9 study texts for students of the helping professions.

Presented a series of lectures at scientific conferences, for the widest public she has published popular articles in the daily press.

She is a member of professional councils of doctoral studies and a member of the Editorial Board of the Contact Journal, the Psychology and its Contexts Journal, Scientific Papers of the Pardubice University, Series D.

From the year 2011 she is a member of the Organizing Committee of e-Conference on Optimization, Education and Data Mining in Science, Engineering and Risk Management.

Assoc.Prof. Ing.VladislavPavlat,CSc.

Education

1940-1948 – high school, final examination 1948
1948-1952 – Economic faculty; state examination; title -Ing.
1956-1960 – doctor studies, title CSc.(= Ph.D.)
1963 – Associated Professor (Docent)

Education abroad

1995 – seminar on financial derivatives at the New York Institute of Finance (diploma)
1990 – US. Securities Exchange Commission in Washington – course organized by US Aid for economists, officially nominated to set up new stock exchanges in former socialist countries
1967 West Germany – Weltwirtschaftsinstitut Kiel; one year scientific stay
1968 and 1969 Great Britain – London School of Economics; six months scientific stay
1964 Luxembourg – Université de Luxembourg, Summer Course on International Economics (diploma „with distinction“)
1963 Luxembourg – Université de Luxembourg, Summer Session on Economics (diploma „avec grande disctinction“)

Employment

2001-2014 University of Finance and Administration in Prague, Faculty of Economic Studie, Department of Finance and Insurance; Associated Professor
1998-2000 financial consultant
1993-1998 chairman of several companies (capital market activities)
1990-1992 Vice-chairman of the Preparatory Committee for setting up of the Prague Stock Exchange
1990-1991 Czechoslovak State Bank, Capital Markets Department; specialist and advisor
1972-1990 Research Institute of Engineering Technology and Economics in Prague; research worker
1952-1970 Prague Economic University; different positions – i. al. Vice-Dean of Faculty for Foreign Trade; Associated Professor; research worker

Other professional activities

1998-2002 member of Chamber of Commerce, Ústí nad Labem (North Bohemia)
1993-1997 Board member of the Prague Stock Exchange and Chairman of the Trading Committee
1992-1997 Chairman of the Board of the company „Capital Market Institute – CMI" (in charge of broker/dealers´ courses and examinations, licenced by the Czech Ministry of Finance
1993-1995 co-founder and Board member of the Czech Association of Securities Dealers
1993-1996 Chief Editor of the journal „Money, Stock Exchange, Capital“
1960-1964 Chief Editor of scientific journal „Czechoslovak Economic Papers“ (published by the Economic Institute of Czechoslovak Academy of Science)

Author of Books (in Czech Language)

„*Globální finanční trhy*“ (*Global Financial Markets*).
Prague: VSFS-EUPRESS, 2013

„ *Regulace a dohled nad kapitálovými trhy*“ (*Capital Market Regulation and Supervision*)
Prague: EUPRESS VŠFS, 2010 (co-author)

„*Kapitálové trhy*“ (*Capital Markets*). 2nd ed.
Prague: Professional Publishing, 2005, editor and co-author

„ *Finanční deriváty*“ (*Financial Derivatives*).
Prague: Magnet Press, 1994

„*Burzy cenných papírů ve světě*“ (*Stock Exchanges in the World*).
Prague: Grada, 1992

„*Světové hospodářství*“ (*World Economy*).
Prague: Svoboda, 1968

Languages

Czech (native speaker), English, German, French, Russian